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Preface

In this book, an attempt is made to investigate the **ELECTRON STATISTICS (ES)** whose importance is well known since the inception of semiconductor science [1–36]. The ES is connected **to many important transport topics of quantum effect devices, namely** *thermoelectric power, Debye screening length, diffusivity-mobility ratio, diffusion coefficient of the minority carriers, the elastic constants, generalized Raman gain, reflection coefficient, hydrostatic piezoresistance coefficient and the gate capacitance, respectively* [37–61]. **It is worth remarking that in this book we shall suggest the methods of experimental determination of the three important transport quantities, namely the Debye screening length, the carrier contribution to the Elastic Constants and the Einstein relation, respectively. The wide applications of Heisenberg's Uncertainty Principle (HUP) in different areas of quantum science are already well established since its inception [62]. In this first of its kind book, we shall use the HUP for finding out the ES for different technologically important quantum materials under different physical conditions in the presence of heavy doping without using the as usual density of states (DOS) function approach in the context for formulating the ES**.

The present monograph explores the ES in heavily doped (HD) nanostructures of nonlinear optical, III-V, II-VI, gallium phosphide, germanium, platinum antimonide, stressed, IV-VI, lead germanium telluride, tellurium, II-V, zinc and cadmium diphosphides, bismuth telluride, III-V quantum confined HD superlattices with graded interfaces, quantum confined HD doping superlattices, quantum confined effective mass superlattices, in the presence of intense light and electric fields and superlattices of HD optoelectronic materials with graded interfaces in addition to other quantized systems. Incidentally, even after forty years of continuous effort, we see that the complete investigation of the ES comprising of the whole set of the HD materials and their quantized counter parts is really a sea and permanently enjoys the domain of impossibility theorems. The ES has different forms for different materials and changes under one-, two- and three-dimensional quantum confinement of the charge carriers. In this context, it may be written that excluding the classic book *Semiconductor Statistics* by J. S. Blakemore (originally published in 1962 by Pergamon press and later on appeared in Dover publication in 1987 with less than 10%

change) which deals only with the ES in bulk and under magnetic quantization in semiconductors having **parabolic energy bands by using the usual DOS function approach and without using the HUP** (in pages 79–116), the available reports on the said areas are not containing the detailed investigations regarding **the ES in the present context by using the HUP**.

It is well known that heavy doping and carrier degeneracy are the keys to unlock the important properties of semiconductors and they are especially instrumental in dictating the characteristics of Ohmic contacts and Schottky contacts, respectively [63–64]. It is an amazing fact that although the HDS have been investigated in the literature, the study of the ES by formulating the corresponding dispersion relations (DRs) of HDS **is still one of the open research problems**. Our method is not at all related to the DOS technique as used in the literature. From the DR, one can obtain the DOS but the DOS technique, as used in the literature cannot generate the dispersion laws. **Therefore, our study is more fundamental than those in the existing literature, because the ES which controls the charge carrier properties of the semiconductor devices is being derived in this monograph in various cases without using the difficult DOS function technique but by directly applying the well-known HUP. This book, containing thirteen chapters, is partially based ongoing research of the different physical properties of the quantized structures from 1980 and we try to present a cross-section preview of the ES for a wide range of HD quantum materials with various DRs under different physical conditions.**

With the advent of radiation science and technology, the study of the optical properties of quantized structures is gaining importance with the assumption that the carrier energy spectra will be unaltered under strong radiation, which is not basically right. The physical properties of semiconductors in the presence of strong light waves which alter the basic dispersion relations have relatively been much less investigated in [63–64] as compared with the cases of other external fields and in opto-electronics the influence of strong light waves is needed for the characterization of the low-dimensional opto-electronic devices. The solo Chap. 1 investigates the ES in bulk specimens of HD Kane type III-V, ternary and quaternary semiconductors under intense radiation by applying the HUP. The same chapter explores the ES in the presence of magnetic quantization, cross-field configuration, quantum wells (QWs), nano-wires (NWs), quantum dots (QDs), magneto size quantization, doping superlattices, magneto doping superlattices, QWHD, NWHD and QDHD effective mass superlattices, magneto QWHD effective mass superlattices, magneto HD effective mass superlattices, QWHD, NWHD and QDHD superlattices with graded interfaces, magneto QWHD superlattices with graded interfaces and magneto HD superlattices with graded interfaces, respectively. **In Chap. 1, we suggest the methods of experimental determinations of the second- and third-order carrier contribution to the elastic constants, the Debye screening length and the Einstein relation, respectively. Additionally, for the purpose of investigating at least one ES-dependent electronic property in this case, we have also investigated the effective electron mass (EEM) which is a very important transport quantity [65–74]**.

The physical properties of ultra-short modern electronic devices have been investigated by using the assumption of invariant carrier dispersion relations under intense electric field, which is not basically right. Chapter 2 of this book investigates the ES by applying the HUP under intense electric field, chronologically for all the specific cases as given in the content of Chap. 2 [75–90]. **It is interesting to note that the EEM depends on the strong electric field** (which is not observed elsewhere) together with the fact that the EEM in the said systems depends on the respective quantum numbers in addition to the Fermi energy the scattering potential and others system constants which are the characteristics features of such hetero-structures.

Chapter 3 deals with the influence of quantum confinement on the ES by applying the HUP in non-parabolic HDS and we study the ES in HDQWs of all the materials as given chronologically in the content of Chap. 3. We will observe that the complex electron dispersion law in HDS instead of real one occurs from the existence of the essential poles in the corresponding electron energy spectrum in the absence of band tails. One important consequence of the HDS forming band tails is that the EEM exists in the forbidden zone, which is impossible without the effect of band tailing. In the absence of band tails, the effective mass in the band gap of semiconductors is infinity. Besides, depending on the type of the unperturbed carrier energy spectrum, the new forbidden zone will appear within the normal energy band gap for HDS. In Chap. 4, the ES for HDNWs of all the materials as stated in the content has respectively been investigated. As a collateral study, we shall observe that the EEM in such NWs becomes a function of size quantum number, the Fermi energy, the scattering potential and other constants of the system which is the intrinsic property of such 1D systems in general. In Chap. 5, the ES for QDs has been studied chronologically for all the materials as stated in the content of Chap. 5.

The various types of semiconductor superlattices (SLs) find extensive device-oriented applications [91–108]. In Chap. 6, we study the ES in doping superlattices (DSL) for all the cases as written in the content of the said chapter. With the advent of MOSFETs, there has been considerable interest in studying the 2D electron transport in such systems [109–128], and in Chap. 7, we study the ES by using HUP in accumulation layers for all the cases as given in the content of Chap. 7. Besides, we have numerically investigated the diffusivity to mobility ratio (DMR) in this context.

The effects of quantizing magnetic field (B) on the band structures of compound semiconductors are profound [129–143], and in Chap. 8, we shall study the ES by applying HUP under magnetic quantization for all the HD materials chronologically as given in the content of the said chapter, and additionally, we investigate at least one ES-dependent property, namely the DMR in this case. It is worth remarking that the effects of crossed electric and quantizing magnetic fields on the transport properties of semiconductors having various band structures have relatively been less investigated as compared with the corresponding magnetic quantization, although the study of the cross-fields is of fundamental importance with respect to the addition of new physics and the related experimental findings in modern quantum effect devices. **Chapter 9 investigates the ES by applying the HUP under cross-field configuration for all the cases as written in the content. This chapter also tells us that the EEM in all the cases is a function of the finite scattering potential,**

the magnetic quantum number, the electric field, the quantizing magnetic field and the Fermi energy even for HD semiconductors whose bulk electrons in the absence of band tails are defined by the parabolic energy bands. As a co-lateral study, we investigate the DMR in this context. Chapter 10 explores the ES in QWs of HD semiconductors under magnetic quantization for all the cases as stated in the content. In Chap. 11, we shall study the ES under cross-field configuration in QWs of HD nonlinear optical, III-V, II-VI, IV-VI and stressed Kane type semiconductors. The cross-fields introduce energy and quantum number-dependent mass anisotropy. Chapter 12 explores the ES in the doping super-lattices chronologically for all the materials as stated in the preface of Chap. 12. The last chapter explores the magneto ES in accumulation layers of different materials.

This book is based on the **'iceberg principle'** [144] and the rest of which will be explored by the researchers of different appropriate fields. Since there is no existing report devoted solely to the study of ES by applying the HUP for HD quantized structures to the best of our knowledge, we earnestly hope that the present book will be a useful reference source for the present and the next generation of the readers and the researchers of materials and allied sciences in general. We have discussed enough regarding the ES in different quantized HD materials although lots of new computer-oriented numerical analysis are being left for the purpose of being computed by the readers, to generate the new graphs and the inferences from them which all together is a sea in itself. Since the production of error free first edition of any book from every point of view is a permanent member of impossibility theorems, therefore in spite of our joint concentrated efforts for couple of years together with the seasoned team of Springer-Verlag, the same stands very true for this monograph also. **Various expressions and few chapters of this book have been appearing for the first time in printed form**. At last, we infer that this book should be useful in graduate courses on materials science, condensed matter physics, solid states electronics, nano-science and technology, and solid-state sciences and devices in many universities and the institutions in addition to both Ph.D. students and researchers in the aforementioned fields.

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For more please go through the link https://scholar.google.co.in/citations?user=ht4qPJsAAAAJ&hl=en.

Chapter 1
The Heisenberg's Uncertainty Principle (HUP) and the Electron Statistics (ES) in Heavily Doped (HD) Kane-type III-V and Opto-Electronic Materials in the Presence of Intense Radiation

1.1 Introduction

Under intense radiation, the different physical properties of quantized nanostructures have been studied [1], using the idea that the $E - k$ relations are not altered under radiation fields which is fundamentally questionable. The electronic properties of electronic materials under strong radiation which changes their dispersion relations have mainly been investigated by Ghatak et al. [2, 3]. The first chapter investigates the ES in HD III-V quantized structures under strong radiation by applying the HUP directly without using the usual density-of-states (DOS) function approach.

We investigate chronologically the ES in accordance with the content of this chapter taking various III-V, ternary, quaternary systems and their low dimensional counterparts which find extensive uses in various device applications [4]. Section 1.4 contains the result and discussions.

1.2 Mathematical Basis

1.2.1 The Bulk Compounds

The carrier dispersion relations (DRs) for III-V and opto-electronic compounds become [5–28]

$$\overline{G}_4 k^2 = I_{11}(E) \tag{1.1a}$$

$$\overline{G}_4 k^2 = E(1 + \alpha E) \tag{1.1b}$$

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and

$$\overline{G}_4 k^2 = E \tag{1.1c}$$

where, $\overline{G}_4 = \frac{\hbar^2}{2m_c}$, $I_{11}(E) = \frac{E(E+E_g)(E+E_g+\Delta)(E_g+\frac{2}{3}\Delta)}{E_g(E_g+\Delta)(E+E_g+\frac{2}{3}\Delta)}$, $\alpha = \frac{1}{E_g}$

and the other notations mean as usual.

In the presence of radiation, Eqs. (1.1a), (1.1b) and (1.1c) assume the forms [29]

$$\overline{G}_4 k^2 = \beta_0(E, \lambda) \tag{1.1d}$$

$$\overline{G}_4 k^2 = \tau_0(E, \lambda) \tag{1.2}$$

and

$$\overline{G}_4 k^2 = \rho_0(E, \lambda) \tag{1.3}$$

where the other notations are written in [29].

Under certain constraints, Eqs. (1.1d), (1.2) and (1.3) become [29]

$$\overline{G}_4 k^2 = U_\lambda I_{11}(E) - P_\lambda \tag{1.4}$$

$$\overline{G}_4 k^2 = t_{1\lambda} E + t_{2\lambda} E^2 - \delta_\lambda \tag{1.5}$$

and

$$\overline{G}_4 k^2 = t_{1\lambda} E - \delta_\lambda \tag{1.6}$$

where the other notations are defined in [29].

The distribution function $F(V)$ here can be written as [6, 7]

$$F(V) = (\pi \eta_g^2)^{-1/2} \exp(-V^2/\eta_g^2) \tag{1.7}$$

Using Eqs. (1.4), (1.5), (1.6) and (1.7) get transformed as [29]

$$\overline{G}_4 k^2 = T_{11}(E, \eta_g, \lambda) \tag{1.8}$$

$$\overline{G}_4 k^2 = T_{21}(E, \eta_g, \lambda) \tag{1.9}$$

and

$$\overline{G}_4 k^2 = T_{31}(E, \eta_g, \lambda) \tag{1.10}$$

The importance of the concept of effective electron mass (EEM) is already well known [30], and using Eqs. (1.8), (1.9) and (1.10), we get

$$m^*(E_{F1}, \eta_g, \lambda)/m_c = \text{R.P.}\,[T_{11}(E_{F1}, \eta_g, \lambda)]' \tag{1.11}$$

$$m^*(E_{F1}, \eta_g, \lambda)/m_c = [T_{21}(E_{F1}, \eta_g, \lambda)]' \tag{1.12}$$

$$m^*(E_{F1}, \eta_g, \lambda)/m_c = [T_{31}(E_{F1}, \eta_g, \lambda)]' \tag{1.13}$$

where, R.P. indicates real part of.

The HUP can be written as

$$\Delta p_i \Delta i \approx A\hbar \tag{1.14}$$

where A is the dimension less constant.

From Eq. (1.14), one can get

$$\Delta k_x \Delta k_y \Delta k_z = \frac{A^3}{\Delta V} \tag{1.15}$$

where $\Delta V = \Delta x \Delta y \Delta z$.

The Paulis' exclusion principle tells us

$$\Delta V = \frac{2}{n_{03D}} \tag{1.16}$$

where n_{03D} is the 3D doping density.

Thus, using Eqs. (1.15) and (1.16), we can write

$$n_{03D} = \frac{2}{C_{3D}}(\Delta k_x \Delta k_y \Delta k_z) \quad \text{where} \quad C_{3D} = A^3 \tag{1.17}$$

If g_v is the valley degeneracy, we can write

$$n_{03D} = \frac{2g_v}{C_{3D}}(\Delta k_x \Delta k_y \Delta k_z) \tag{1.18}$$

For two and one dimensions, we get

$$n_{02D} = \frac{2g_v}{C_{2D}}(\Delta k_x \Delta k_y) \tag{1.19}$$

and

$$n_{01D} = \frac{2g_v}{C_{1D}}(\Delta k_x) \tag{1.20}$$

and where n_{02D} is 2D doping density, n_{01D} is the 1D doping density and C_{2D} and C_{1D} are two dimensionless constants.

We can write

$$V(E_F) = (\Delta k_x \Delta k_y \Delta k_z) \tag{1.21}$$

From Eq. (1.8), we observe that the constant energy k-space is a sphere and we can write

$$V(E_{F1}) = \frac{4\pi}{3}[\overline{G}_4]^{3/2}\text{R.P.}\,[T_{11}(E_{F1}, \eta_g\lambda)]^{\frac{3}{2}} \tag{1.22}$$

Thus, the ES can be written as

$$n_0 = \frac{8\pi g_v}{3C_{3D}}[\overline{G}_4]^{3/2}\text{R.P.}\,[T_{11}(E_{F1}, \eta_g\lambda)]^{3/2} \tag{1.23}$$

Under the substitution $C_{3D} = 1$, Eq. (1.23) can be written as

$$n_0 = \overline{G}_1 R.P.[T_{11}(E_{F1}, \eta_g, \lambda)]^{3/2} \tag{1.24}$$

where, $\overline{G}_1 = \frac{8\pi g_v}{3}[\frac{\hbar^2}{2m_c}]^{3/2}$.

Similarly, the use of Eqs. (1.9) and (1.10) leads to the expressions of ES as

$$n_0 = \overline{G}_1[T_{21}(E_{F1}, \eta_g, \lambda)]^{3/2} \tag{1.25}$$

$$n_0 = \overline{G}_1[T_{31}(E_{F1}, \eta_g, \lambda)]^{3/2} \tag{1.26}$$

When $\eta_g \to 0$, the Eqs. (1.11)–(1.13) gets simplified as

$$m^*(E_{F2}, \lambda)/m_c = [\beta_0(E_{F2}, \lambda)]' \tag{1.27}$$

$$m^*(E_{F2}, \lambda)/m_c = [\tau_0(E_{F2}, \lambda)]' \tag{1.28}$$

$$m^*(E_{F2}, \lambda)/m_c = [\rho_0(E_{F2}, \lambda)]' \tag{1.29}$$

where E_{F2} is the corresponding Fermi energy.

The ES assumes the forms

$$n_0 = \overline{G}_2\,[\beta_0(E_{F2}, \lambda)]^{3/2} \tag{1.30}$$

$$n_0 = \overline{G}_2\,[\tau_0(E_{F2}, \lambda)]^{3/2} \tag{1.31}$$

$$n_0 = \overline{G}_2[\rho_0(E_{F2}, \lambda)]^{3/2} \tag{1.32}$$

where, $\overline{G}_2 = \frac{g_v}{3\pi^2}(\frac{2m_c}{\hbar^2})^{3/2}$.

When $I \rightarrow 0$ Eqs. (1.27)–(1.29) get simplified to the well—known forms as [31, 32]

$$m^*(E_{F3})/m_c{=}[I_{11}(E_{F3})]' \tag{1.33}$$

$$m^*(E_{F3})/m_c{=}(1 + 2\alpha E_{F3})' \tag{1.34}$$

$$m^*(E_{F3})/m_c{=}\,1 \tag{1.35}$$

The ES assumes the well-known forms as [30]

$$n_0 = \overline{G}_2\;[I_{11}(E_{F3})]^{3/2} \tag{1.36}$$

$$n_0 = \overline{G}_2\,[E_{F3}(1 + \alpha E_{F3})]^{3/2} \tag{1.37}$$

$$n_0 = \overline{G}_2[E_{F3}]^{3/2}. \tag{1.38}$$

1.2.2 The Magnetic Quantization

Here, we can write

$$\pi k_s^2 = \frac{2\pi eB}{\hbar}\left(n + \frac{1}{2}\right) \tag{1.39}$$

where $k_s^2 = k_x^2 + k_y^2$.

(i) Using Eqs. (1.8) and (1.39), the DR becomes

$$T_{11}(E, \eta_g, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m_c} \tag{1.40}$$

where, $\omega_0 = (eB/m_c)$.

The EEM becomes

$$m^*(E_{F4}, \eta_g, \lambda)/m_c = \text{R.P.}\{T_{11}(E_{_{F4}}, \eta_g, \lambda)\}' \tag{1.41}$$

where E_{F4} is the corresponding Fermi energy.

By using the HUP, the ES becomes

$$n_0 = \frac{4g_v eB}{C_B\hbar}\sum_{n=0}^{n_{\max}} \Delta k_z \tag{1.42}$$

where C_B is a constant.

From Eq. (1.40), we get

$$T_{11}(E_{F4}, \eta_g, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2(\Delta k_z)^2}{2m_c} \tag{1.43}$$

Substituting $C_B = 4\pi^2$ and eliminating Δk_z between Eqs. (1.42) and (1.43), the ES can be can be expressed as

$$n_0 = \overline{G}_3\,\text{R.P.}\sum_{n=0}^{n_{\max}}[\{T_{11}(E_{F4}, \eta_g, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0\}^{1/2}] \tag{1.44}$$

where, $\overline{G}_3 = \frac{Bg_v|e|\sqrt{2m_c}}{\pi^2\hbar^2}$.

Using Eqs. (1.39) and (1.1d), the magneto-dispersion law when $\eta_g \to 0$ is given by

$$\beta_0(E, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \overline{G}_4 k_z^2 \tag{1.45}$$

The EEM becomes

$$m^*(E_{F5}, \lambda)/m_c = \{\beta_0(E_{F5}, \lambda)\}' \tag{1.46}$$

The ES assumes the form

$$n_0 = \overline{G}_3\sum_{n=0}^{n_{\max}}\left[\left\{\beta_0(E_{F5}, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0\right\}^{1/2}\right] \tag{1.47}$$

When additionally $I \to 0$, the ES can be expressed from Eq. (1.47) as

$$n_0 = \bar{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ I_{11}(E_{F6}) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.48}$$

(ii) Using Eqs. (1.9) and (1.39), the corresponding DR becomes

$$T_{21}(E, \eta_g, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m_c} \tag{1.49}$$

The EEM gets simplified as

$$m^*(E_{F4}, \eta_g, \lambda)/m_c = \{T_{21}(E_{F4}, \eta_g, \lambda)\}' \tag{1.50}$$

The ES is given by

$$n_0 = \overline{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ T_{21}(E_{F4}, \eta_g, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.51}$$

Using Eqs. (1.2) and (1.39), the DR becomes

$$\tau_0(E, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m_c} \tag{1.52}$$

Using Eq. (1.52), the EEM becomes

$$m^*(E_{F5}, \lambda)/m_c = \{\tau_0(E_{F5}, \lambda)\}' \tag{1.53}$$

The ES is given by

$$n_0 = \overline{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ \tau_0(E_{F5}, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.54}$$

When additionally $I \to 0$, the ES can be expressed from Eq. (1.54) as

$$n_0 = \overline{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ E_{F6}(1 + \alpha E_{F6}) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.55}$$

(iii) Using Eqs. (1.3) and (1.39), the DR becomes

$$T_{31}(E, \eta_g, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m_c} \tag{1.56}$$

The corresponding EEM becomes

$$m^*(E_{F4}, \eta_g, \lambda)/m_c = \{T_{31}(E_{F4}, \eta_g, \lambda)\}' \tag{1.57}$$

The ES can be written as

$$n_0 = \bar{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ T_{31}(E_{F4}, \eta_g, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.58}$$

Using Eqs. (1.11) and (1.39), the DR becomes

$$\rho_0(E, \lambda) = (n + \frac{1}{2})\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m_c} \tag{1.59}$$

The EEM becomes

$$m^*(E_{F5}, \lambda)/m_c = \{\rho_0(E_{F5}, \lambda)\}' \tag{1.60}$$

The ES is given by

$$n_0 = \bar{G}_3 \sum_{n=0}^{n_{\max}} \left[\left\{ \rho_0(E_{F5}, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right\}^{1/2} \right] \tag{1.61}$$

When additionally $I \rightarrow 0$, the ES assumes the well-known forms [16, 29] which can be expressed from Eq. (1.61) as

$$n_0 = \overline{G}_3 \sum_{n=0}^{n_{\max}} \left[E_{F6} - \left(n + \frac{1}{2}\right)\hbar\omega_0 \right]^{\frac{1}{2}}. \tag{1.62}$$

1.2.3 The Cross Fields Configuration

(i) Using Eq. (1.8), the DR assumes the form

$$\begin{aligned} T_{11}(E, \eta_g, \lambda) = &\left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} \\ &- \frac{E_0}{B}\hbar k_y \{T_{11}(E, \eta_g, \lambda)\}' \\ &- \left\{ \frac{m_c E_0^2 [\{T_{11}(E, \eta_g, \lambda)\}']^2}{2B^2} \right\} \end{aligned} \tag{1.63}$$

where E_0 the electric field along x direction.

Using HUP, the ES assumes the form

$$n_0 = X_1 \text{R.P.}\,[R_1] \tag{1.64}$$

where

$$\begin{aligned} X_1 = \left(\frac{2g_v B\sqrt{2m_c}}{3L_x\pi^2\hbar^2 E_0}\right)\sum_{n=0}^{n_{\max}}, R_1 = [[T_{11}(E_{F7},\eta_g,\lambda) \\ -(n+\frac{1}{2})\hbar\omega_0 - \frac{m_c E_0^2}{2B^2}[\{T_{11}(E_{F7},\eta_g,\lambda)\}']^2 \\ +|e|E_0 L_x[\{T_{11}(E_{F7},\eta_g,\lambda)\}']]^{3/2} \\ -[T_{11}(E_{F7},\eta_g,\lambda) - (n+\frac{1}{2})\hbar\omega_0 \\ -\frac{m_c E_0^2}{2B^2}[\{T_{11}(E_{F7},\eta_g,\lambda)\}']^2]^{3/2}][\{T_{11}(E_{F7},\eta_g,\lambda)\}']^{-1} \end{aligned}$$

and E_{F7} is the corresponding Fermi energy.

When $\eta_g \to 0$, Eq. (1.63) assumes the form

$$\begin{aligned} \beta_0(E,\lambda) = \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} \\ -\frac{E_0}{B}\hbar k_y\{\beta_0(E,\lambda)\}' - \left\{\frac{m_c E_0^2[\{\beta_0(E,\lambda)\}']^2}{2B^2}\right\} \end{aligned} \tag{1.65}$$

Using HUP, the ES assumes the form

$$n_0 = X_1[R_2] \tag{1.66}$$

where

$$\begin{aligned} R_2 \equiv [[\beta_0(E_{F8},\lambda) - \left(n+\frac{1}{2}\right)\hbar\omega_0 \\ -\frac{m_c E_0^2}{2B^2}[\{\beta_0(E_{F8},\lambda)\}']^2 \\ +|e|E_0 L_x[\{\beta_0(E_{F8},\lambda)\}']]^{3/2} \\ -[\beta_0(E_{F8},\lambda) - (n+\frac{1}{2})\hbar\omega_0 \\ -\frac{m_c E_0^2}{2B^2}[\{\beta_0(E_{F8},\lambda)\}']^2]^{3/2}][\{\beta_0(E_{F8},\lambda)\}']^{-1}] \end{aligned}$$

(ii) Using Eq. (1.9), the DR becomes

$$T_{21}(E,\eta_g,\lambda)=\left(n+\frac{1}{2}\right)\hbar\omega_0+\frac{[\hbar k_z(E)]^2}{2m_c}-\frac{E_0}{B}\hbar k_y\{T_{21}(E,\eta_g,\lambda)\}'-\left\{\frac{m_c E_0^2[\{T_{21}(E,\eta_g,\lambda)\}']^2}{2B^2}\right\} \tag{1.67}$$

The ES is given by

$$n_0=X_1[R_3] \tag{1.68}$$

where

$$R_3=[[T_{21}(E_{F7},\eta_g,\lambda)-\left(n+\frac{1}{2}\right)\hbar\omega_0-\frac{m_c E_0^2}{2B^2}[\{T_{21}(E_{F7},\eta_g,\lambda)\}']^2+|e|E_0L_x[\{T_{21}(E_{F7},\eta_g,\lambda)\}']]^{3/2}-[T_{21}(E_{F7},\eta_g,\lambda)-(n+\frac{1}{2})\hbar\omega_0-\frac{m_c E_0^2}{2B^2}[\{T_{21}(E_{F7},\eta_g,\lambda)\}']^2]^{3/2}][\{T_{21}(E_{F7},\eta_g,\lambda)\}']^{-1}$$

When $\eta_g\to 0$, Eq. (1.67) assumes the form

$$\tau_0(E,\lambda)=\left(n+\frac{1}{2}\right)\hbar\omega_0+\frac{[\hbar k_z(E)]^2}{2m_c}-\frac{E_0}{B}\hbar k_y\{\tau_0(E,\lambda)\}'-\left\{\frac{m_c E_0^2[\{\tau_0(E,\lambda)\}']^2}{2B^2}\right\} \tag{1.69}$$

Using HUP, the ES assumes the form

$$n_0=X_1[R_4] \tag{1.70}$$

where

$$R_4\equiv[[\tau_0(E_{F8},\lambda)-\left(n+\frac{1}{2}\right)\hbar\omega_0-\frac{m_c E_0^2}{2B^2}[\{\tau_0(E_{F8},\lambda)\}']^2$$

$$+ |e|E_0 L_x[\{\tau_0(E_{F8}, \lambda)\}']]^{3/2}$$
$$- [\tau_0(E_{F8}, \lambda) - (n + \frac{1}{2})\hbar\omega_0$$
$$- \frac{m_c E_0^2}{2B^2}[\{\tau_0(E_{F8}, \lambda)\}']^2]^{3/2}][\{\tau_0(E_{F8}, \lambda)\}']^{-1}]$$

(iii) Using Eq. (1.10), the DR assumes the form

$$T_{31}(E, \eta_g, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} - \frac{E_0}{B}\hbar k_y\{T_{31}(E, \eta_g, \lambda)\}' - \left\{\frac{m_c E_0^2[\{T_{31}(E, \eta_g, \lambda)\}']^2}{2B^2}\right\} \tag{1.71}$$

The ES is given by

$$n_0 = X_1[R_5] \tag{1.72}$$

where

$$R_5 \equiv [[T_{31}(E_{F7}, \eta_g, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{m_c E_0^2}{2B^2}[\{T_{31}(E_{F7}, \eta_g, \lambda)\}']^2 + |e|E_0 L_x[\{T_{31}(E_{F7}, \eta_g, \lambda)\}']]^{3/2} - [T_{31}(E_{F7}, \eta_g, \lambda) - \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{m_c E_0^2}{2B^2}[\{T_{31}(E_{F7}, \eta_g, \lambda)\}']^2]^{3/2}][\{T_{31}(E_{F7}, \eta_g, \lambda)\}']^{-1}$$

When $\eta_g \to 0$, Eq. (1.71) assumes the form

$$\rho_0(E, \lambda) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} - \frac{E_0}{B}\hbar k_y\{\rho_0(E, \lambda)\}' - \left\{\frac{m_c E_0^2[\{\rho_0(E, \lambda)\}']^2}{2B^2}\right\} \tag{1.73}$$

The ES is given by

$$n_0 = X_1[R_6] \tag{1.74}$$

where

$$\begin{aligned} R_6 \equiv [[\rho_0(E_{F8},\lambda) - \left(n+\frac{1}{2}\right)\hbar\omega_0 \\ - \frac{m_c E_0^2}{2B^2}[\{\rho_0(E_{F8},\lambda)\}']^2 \\ + |e|E_0 L_x[\{\rho_0(E_{F8},\lambda)\}']]^{3/2} \\ - [\rho_0(E_{F8},\lambda) - (n+\frac{1}{2})\hbar\omega_0 \\ - \frac{m_c E_0^2}{2B^2}[\{\rho_0(E_{F8},\lambda)\}']^2]^{3/2}][\{\rho_0(E_{F8},\lambda)\}']^{-1} \end{aligned}$$

(iv) When $\eta_g \to 0$ and $I \to 0$, Eq. (1.65) assumes the form

$$\begin{aligned} I_{11}(E) = \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} \\ - \frac{E_0}{B}\hbar k_y\{I_{11}(E)\}' - \frac{m_c E_0^2[\{I_{11}(E)\}']^2}{2B^2} \end{aligned} \tag{1.75}$$

The ES in this case assume the forms

$$n_0 = X_1[R_7] \tag{1.76}$$

where

$$\begin{aligned} R_7 \equiv [[I_{11}(E_{F9}) - (n+\frac{1}{2})\hbar\omega_0 \\ - \frac{m_c E_0^2}{2B^2}[\{I_{11}(E_{F9})\}']^2 \\ + |e|E_0 L_x[\{I_{11}(E_{F9})\}']]^{3/2} \\ - [I_{11}(E_{F9}) - (n+\frac{1}{2})\hbar\omega_0 \\ - \frac{m_c E_0^2}{2B^2}[\{I_{11}(E_{F9})\}']^2]^{3/2}][\{I_{11}(E_{F9})\}']^{-1}] \end{aligned}$$

and E_{F9} is the corresponding Fermi energy.

(v) Under the condition $\Delta \gg E_g$, Eq. (1.75) gets simplified as [29]

$$\begin{aligned} E(1+\alpha E) = \left(n+\frac{1}{2}\right)\hbar\omega_0 - \frac{E_0}{B}\hbar k_y(1+2\alpha E) \\ - \frac{m_c E_0^2}{2B^2}(1+2\alpha E)^2 + \frac{[\hbar k_z(E)]^2}{2m_c} \end{aligned} \tag{1.77}$$

The ES is given by

$$n_0 = X_1[R_8] \tag{1.78}$$

where

$$\begin{aligned} R_8 \equiv [[E_{F10}(1+\alpha E_{F10}) - \left(n+\frac{1}{2}\right)\hbar\omega_0 \\ + |e|E_0 L_x(1+2\alpha E_{F10}) \\ - \frac{m_c E_0^2}{2B^2}(1+2\alpha E_{F10})^2]^{3/2} \\ - [E_{F10}(1+\alpha E_{F10}) - \left(n+\frac{1}{2}\right)\hbar\omega_0 \\ - \frac{m_c E_0^2}{2B^2}(1+2\alpha E_{F10})^2]^{3/2}][1+2\alpha E_{F10}]^{-1} \end{aligned}$$

and E_{F10} is the corresponding Fermi energy.

1.2.4 The Quantum Wells

(i) For Eq. (1.8), the 2D DR in HDQWs is given by

$$\overline{G}_4 k_s^2 + \overline{G}_5 = T_{11}(E, \eta_g, \lambda) \tag{1.79}$$

where, $\overline{G}_5 = \frac{\hbar^2}{2m_c}(\frac{n_z\pi}{d_z})^2$.

Using Eq. (1.79), the EEM assumes the form

$$m^*(E_{F10}, n_z, \lambda)/m_c = \text{R.P.}\{T_{11}(E_{F10}, \eta_g, \lambda)\}' \tag{1.80}$$

where E_{F10} is the corresponding Fermi energy.

Using (1.79) and taking the contribution of all the sub-bands by summing and taking $C_{2D} = 1$, the ES is given by

$$n_0 = (\overline{G}_6) R.P. \sum_{n_z=1}^{n_{z\max}} [T_{11}(E_{F10,} n_z, \lambda) - \overline{G}_5] \tag{1.81}$$

where, $\overline{G}_6 = \frac{m_c g_v}{\pi\hbar^2}$.

For $\eta_g \to 0$, the 2D DR can be expressed from Eq. (1.79) as

$$\overline{G}_4 k_s^2 + \overline{G}_5 = \beta_0(E, \lambda) \tag{1.82a}$$

Using (1.81), we get

$$m^*(E_{F11}, n_z, \lambda)/\, m_c = \{\beta_0(E_{F11}, \lambda)\}' \tag{1.82b}$$

The ES is given by

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [\beta_0(E_{F11}, n_z, \lambda) - \overline{G}_5] \tag{1.83}$$

Under the conditions $\eta_g \to 0$ and $I \to 0$, from Eq. (1.81) we get

$$\overline{G}_4 k_s^2 + \overline{G}_5 = I_{11}(E) \tag{1.84}$$

Using (1.84), the EMM is given by

$$m^*(E_{F12})/m_c = \{I_{11}(E_{F12})\}' \tag{1.85}$$

where E_{F12} is the corresponding Fermi energy.

The ES corresponding to Eq. (1.84) assumes the form

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{53}(E_{F12}, n_z)] \tag{1.86}$$

where, $T_{53}(E_{F12}, n_z) \equiv [I_{11}(E_{F12}) - \overline{G}_5]$.

(ii) The 2D DR, EEM and the ES in HDQWs for Eq. (1.9) can, respectively, be written for $I \neq 0$ as

$$\overline{G}_4 k_s^2 + \overline{G}_5 = T_{21}(E, \eta_g, \lambda) \tag{1.87}$$

$$m^*(E_{F10,} n_z, \lambda)/\, m_c = \{T_{21}(E_{F10}, \eta_g, \lambda)\}' \tag{1.88}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{21}(E_{F10,} n_z, \lambda) - \overline{G}_5] \tag{1.89}$$

Equations (1.87), (1.88) and (1.89) for $\eta_g \to 0$ get simplified as

$$\overline{G}_4 k_s^2 + \overline{G}_5 = \tau_0(E, \lambda) \tag{1.90}$$

$$m^*(E_{F11}, n_z, \lambda)/m_c = \{\tau_0(E_{F11}, \lambda)\}' \tag{1.91}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [\tau_0(E_{F11}, n_z, \lambda) - \overline{G}_5] \tag{1.92}$$

Equations (1.90), (1.91) and (1.92) for $I \to 0$ become

$$E(1 + \alpha E) = \overline{G}_4 k_s^2 + \overline{G}_5 \tag{1.93}$$

$$m^*(E_{F12})/m_c = (1 + 2\alpha E_{F12}) \tag{1.94}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} (1 + 2\alpha E_{n_{z3}})[[E_{F12} - E_{n_{z3}}] + \alpha[E_{F12} - E_{n_{z3}}]^2] \tag{1.95}$$

(iii) The 2D DR, EEM and the ES in QWs for Eq. (1.10) can be written as

$$\overline{G}_4 k_s^2 + \overline{G}_5 = T_{31}(E, \eta_g, \lambda) \tag{1.96}$$

$$m^*(E_{F10}, n_z, \lambda)/m_c = \{T_{31}(E_{F10}, \eta_g, \lambda)\}' \tag{1.97}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{31}(E_{F10,} n_z, \lambda) - \overline{G}_5] \tag{1.98}$$

Equations (1.96), (1.97) and (1.98) under the condition $\eta_g \to 0$ get simplified as

$$\overline{G}_4 k_s^2 + \overline{G}_5 = \rho_0(E, \lambda) \tag{1.99}$$

$$m^*(E_{F11}, n_z, \lambda)/m_c = \{\rho_0(E_{F11}, \lambda)\}' \tag{1.100}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [\rho_0(E_{F11}, n_z, \lambda) - \overline{G}_5] \tag{1.101}$$

Equations (1.99), (1.100) and (1.101) under the condition $I \to 0$ get simplified as

$$E = \overline{G}_4 k_s^2 + \overline{G}_5 \tag{1.102}$$

$$m^*(E_{F12})/m_c = 1 \tag{1.103}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [(E_{F12} - E_{n_{z_{33}}})]. \tag{1.104}$$

1.2.5 The Doping Superlattices

(i) The DR in doping super lattices for Eq. (1.8) becomes

$$T_{11}(E, \eta_g, \lambda) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{91\mathrm{HD}}(E, \eta_g, \lambda) + \overline{G}_4 k_s^2 \tag{1.105}$$

where $\omega_{91\mathrm{HD}}(E, \eta_g, \lambda) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc} T'_{11}(E,\eta_g,\lambda) m_c})^{1/2}$.

From Eq. (1.105), the EEM becomes

$$(m^*(E_{F13,}\eta_g, \lambda, n_i)/m_c) = \mathrm{R.P.}\{M_{40\mathrm{HD}}(E_{F13,}\eta_g, \lambda, n_i)\}' \tag{1.106}$$

where, $M_{40\mathrm{HD}}(E_{F13,}\eta_g, \lambda, n_i) = \{T_{11}(E_{F13}, \eta_g, \lambda) - (n_i + \frac{1}{2})\hbar\omega_{91\mathrm{HD}}(E_{F13}, \eta_g, \lambda)\}$.

The ES becomes

$$n_s = \overline{G}_6 \mathrm{R.P.} \sum_{n_i=0}^{n_{i\max}} [M_{40\mathrm{HD}}(E_{F13}, \eta_g, \lambda)] \tag{1.107}$$

Under the condition $\eta_g \to 0$, Eqs. (1.105), (1.106) and (1.107) get simplified as

$$\beta_0(E, \lambda) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{911}(E, \lambda) + \overline{G}_4 k_s^2 \tag{1.108}$$

$$m^*(E_{F14}, \lambda, n_i)/m_c = \{M_{401}(E_{F14}, \lambda, n_i)\}' \tag{1.109}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [M_{401}(E_{F14}, \lambda)] \tag{1.110}$$

where, $\omega_{911}(E, \lambda) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc}\beta'_0(E,\lambda)m_c})^{1/2}$ and $M_{401}(E_{F14}, \lambda, n_i) = \{\beta_0(E_{F14}, \lambda) - (n_i + \frac{1}{2})\hbar\omega_{911}(E_{F14}, \lambda)\}$ and E_{F14} is the corresponding Fermi energy

Under the conditions $\eta_g \to 0$ and $I \to 0$, Eqs. (1.108), (1.109) and (1.110) get simplified as

$$I_{11}(E) = \left(n_i + \frac{1}{2}\right)\hbar\omega_9(E) + \overline{G}_4 k_s^2 \tag{1.111}$$

$$m^*(E_{F15}, n_i)/m_c = R_{82}(E_{F15}, n_i) \tag{1.112}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [T_{83}(E_{F15}, n_i)] \tag{1.113}$$

where, $\omega_9(E) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc} I'_{11}(E) m_c})^{1/2}$

$$R_{82}(E, n_i) = \left\{[I_{11}(E)]' - \left(n_i + \frac{1}{2}\right)h[\omega_9(E)]'\right\},$$
$$T_{83}(E_{F15}, n_i) \equiv \left[I_{11}(E_{F15}) - \left(n_i + \frac{1}{2}\right)\hbar\omega_9(E_{F15})\right]$$

and E_{F15} is the corresponding Fermi energy.

(ii) For Eq. (1.9), the DR, EEM and ES assume the forms

$$T_{21}(E, \eta_g, \lambda) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{92\text{HD}}(E, \eta_g, \lambda) + \overline{G}_4 k_s^2 \tag{1.114}$$

$$m^*(E_{F13,}\eta_g, \lambda, n_i)/m_c = \{M_{41\text{HD}}(E_{F13}, \eta_g, \lambda, n_i)\}' \tag{1.115}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [M_{41\text{HD}}(E_{F13}, \eta_g, \lambda)] \tag{1.116}$$

where $\omega_{92\text{HD}}(E, \eta_g, \lambda) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc} T'_2(E,\eta_g,\lambda) m_c})^{1/2}$ and

$$M_{41\text{HD}}(E_{F13,}\eta_g, \lambda, n_i) = \{T_{21}(E_{F13}, \eta_g, \lambda) - \left(n_i + \frac{1}{2}\right)\hbar\omega_{92\text{HD}}(E_{F13}, \eta_g, \lambda)\},$$

Under the condition $\eta_g \to 0$, Eqs. (1.114), (1.115) and (1.116) get simplified as,

$$\tau_0(E, \lambda) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{912}(E, \lambda) + \overline{G}_4 k_s^2 \tag{1.117}$$

$$m^*(E_{F14}, \lambda, n_i)/m_c = \{M_{402}(E_{F14}, \lambda, n_i)\}' \tag{1.118}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [M_{402}(E_{F14}, \lambda)] \tag{1.119}$$

where

$$\omega_{912}(E, \lambda) = \left(\frac{n_0 |e|^2}{\varepsilon_{sc} \tau_0'(E, \lambda) m_c} \right)^{1/2} \quad \text{and}$$

$$M_{402}(E_{F14}, \lambda, n_i) = \left\{ \tau_0(E_{F14}, \lambda) - \left(n_i + \frac{1}{2} \right) \hbar \omega_{912}(E_{F14}, \lambda) \right\}$$

Under the conditions $\eta_g \rightarrow 0$ and $I \rightarrow 0$, Eqs. (1.117), (1.18) and (1.119) get simplified as,

$$E(1 + \alpha E) = \left(n_i + \frac{1}{2} \right) \hbar \omega_{10}(E) + \overline{G}_4 k_s^2 \tag{1.120}$$

$$m^*(E_{F15}, n_i)/m_c = R_{83}(E_{F15}, n_i) \tag{1.121}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\,\max}} [T_{832}(E_{F15}, n_i)] \tag{1.122}$$

$$R_{82}(E, n_i) \equiv \left\{ [(1 + 2\alpha E)] - \left(n_i + \frac{1}{2} \right) \hbar [\omega_{10}(E)]' \right\}$$

and $T_{832}(E_{F15}, n_i) \equiv [E_{F15}(1 + \alpha E_{F15}) - (n_i + \frac{1}{2}) \hbar \omega_{10}(E_{F15})]$.

(iii) For Eq. (1.10), the DR, EEM and ES become

$$T_{31}(E, \eta_g, \lambda) = \left(n_i + \frac{1}{2} \right) \omega_{93\text{HD}}(E, \eta_g, \lambda) + \overline{G}_4 k_s^2 \tag{1.123}$$

$$m^*(E_{F13,} \eta_g, \lambda, n_i) = m_c \{ M_{42\text{HD}}(E_{F13}, \eta_g, \lambda, n_i) \}' \tag{1.124}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [M_{42\text{HD}}(E_{F13}, \eta_g, \lambda)] \tag{1.125}$$

where $\omega_{93HD}(E, \eta_g, \lambda) \equiv (\frac{n_0 |e|^2}{\varepsilon_{sc} T_3'(E, \eta_g, \lambda) m_c})^{1/2}$.

and $M_{42\text{HD}}(E_{F13}, \eta_g, \lambda, n_i) = \{ T_{31}(E_{F13}, \eta_g, \lambda) - (n_i + \frac{1}{2}) \hbar \omega_{93HD}(E_{F13}, \eta_g, \lambda) \}$.

Under the condition $\eta_g \rightarrow 0$, Eqs. (1.126), (1.127) and (1.128) get simplified as,

$$\rho_0(E,\lambda) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{913}(E,\lambda) + \overline{G}_4 k_s^2 \tag{1.126}$$

$$m^*(E_{F15},\lambda,n_i)/m_c = \{M_{403}(E_{F15},\lambda,n_i)\}' \tag{1.127}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\max}} [M_{403}(E_{F15},\lambda)] \tag{1.128}$$

where $\omega_{913}(E,\lambda) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc}\rho_0'(E,\lambda)m_c})^{1/2}$.

and $M_{403}(E_{F15},\lambda,n_i) = \{\rho_0(E_{F15},\lambda) - (n_i + \frac{1}{2})\hbar\omega_{913}(E_{F15},\lambda)\}$.

Under the conditions $\eta_g \to 0$ and $I \to 0$, Eqs. (1.117), (1.18) and (1.119) get simplified as.

$$E = \left(n_i + \frac{1}{2}\right)\hbar\omega_{11} + \overline{G}_4 k_s^2 \tag{1.129}$$

$$m^*(E_{F15},n_i)/m_c = 1 \tag{1.130}$$

$$n_s = \overline{G}_6 \sum_{n_i=0}^{n_{i\,\max}} (E_{F15} - E_{4ni}) \tag{1.131}$$

where, $\omega_{11} \equiv (\frac{n_0|e|^2}{\varepsilon_{sc}m_c})^{1/2}$.

1.2.6 The Quantum Dots

(i) The DR in quantum dots for Eq. (1.8) can be shown as

$$\overline{G}_7 = T_{11}(E_{17,1},\eta_g,\lambda) \tag{1.132}$$

$$n_0 = \overline{G}_8 \text{R.P.}\,[F_{-1}(\eta_{126})] \tag{1.133}$$

where, $\overline{G}_7 = \frac{\hbar^2(n_z\pi/d_z)^2}{2m_c} + \frac{\hbar^2(n_y\pi/d_y)^2}{2m_c} + \frac{\hbar^2(n_x\pi/d_x)^2}{2m_c}$.

$$\overline{G}_8 = \frac{2g_v}{d_x d_y d_z} \sum_{n_x=1}^{n_{x\max}} \sum_{n_y=1}^{n_{y\max}} \sum_{n_z=1}^{n_{z\max}} \quad \text{and} \quad \eta_{126} = \frac{E_{F16} - E_{17.1}}{k_B T}.$$

When $\eta_g \to 0$, the DR and the ES become

$$\beta_0(E_{17,2}, \lambda) = \overline{G}_7 \tag{1.134}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1261}) \tag{1.135}$$

where,

$$\eta_{1261} = \frac{E_{F17} - E_{17,2}}{k_B T}$$

When $\eta_g \to 0$ and $I \to 0$, the DR and the ES can, respectively, be written as

$$I_{11}(E_{17,3}) = \overline{G}_7 \tag{1.136}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1262}) \tag{1.137}$$

where

$$\eta_{1262} = \frac{E_{F18} - E_{17,3}}{k_B T}$$

(ii) The DR and the ES for Eq. (1.9) can, respectively, be written as

$$\overline{G}_7 = T_{21}(E_{17,4}, \eta_g, \lambda) \tag{1.138}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{127}) \tag{1.139}$$

where
$\eta_{127} = \frac{E_{F16} - E_{17.4}}{k_B T}$ and $E_{17.4}$ is the sub-band energy.
When $\eta_g \to 0$, the DR and the ES can, respectively, be written as

$$\tau_0(E_{17,5}, \lambda) = \overline{G}_7 \tag{1.140}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1264}) \tag{1.141}$$

where

$$\eta_{1264} = \frac{E_{F17} - E_{17,5}}{k_B T}$$

When $\eta_g \to 0$ and $I \to 0$, the DR and the ES become

$$E_{17,6}(1 + \alpha E_{17,6}) = \overline{G}_7 \tag{1.142}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1265}) \tag{1.143}$$

where

$$\eta_{1265} = (k_B T)^{-1}(E_{F18} - E_{17,6})$$

(iii) The DR and the ES for Eq. (1.10) can, respectively, be written as

$$\overline{G}_7 = T_{31}(E_{17,7}, \eta_g, \lambda) \tag{1.144}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1275}) \tag{1.145}$$

where

$$\eta_{1275} = (k_B T)^{-1}(E_{F16} - E_{17,7})$$

When $\eta_g \to 0$, the DR and the ES become

$$\rho_0(E_{17,8}, \lambda) = \overline{G}_7 \tag{1.146}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{1265}) \tag{1.147}$$

where

$$\bar{\eta}_{1265} = (k_B T)^{-1}(E_{F17} - E_{17,8})$$

For $\eta_g \to 0$ and $I \to 0$, the sub-band energy $E_{17,9}$ and n_0 can, respectively, be written as

$$E_{17,9} = \overline{G}_7 \tag{1.148}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{1266}) \tag{1.149}$$

where

$$\eta_{1266} = (k_B T)^{-1}(E_{F18} - E_{17,9}).$$

1.2.7 The Magneto-Size Quantization

(i) From Eq. (1.79), $E_{17,10}$ and n_0 become

$$\overline{G}_9 = T_{11}(E_{17,10}, \eta_g, \lambda) \tag{1.150}$$

$$n_0 = \overline{G}_{10}\text{R.P. } F_{-1}(\eta_{128}) \tag{1.151}$$

where, $\overline{G}_9 = (n + \frac{1}{2})\hbar\omega_0 + \frac{\hbar^2}{2m_c}(\frac{n_z\pi}{d_z})^2$, $\overline{G}_{10} = \frac{eBg_v}{\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{n_z=1}^{n_{z\max}}$, $\eta_{128} = (k_BT)^{-1}(E_{F19} - E_{17,10})$ and E_{F19} is the corresponding Fermi energy.

Under the condition $\eta_g \to 0$, Eqs. (1.150) and (1.151) assume the forms

$$\overline{G}_9 = \beta_0(E_{17,11}, \lambda) \tag{1.152}$$

$$n_0 = \overline{G}_{10}F_{-1}(\eta_{129}) \tag{1.153}$$

where

$\eta_{129} = (k_BT)^{-1}(E_{F20} - E_{17,11})$, E_{F20} is the corresponding Fermi energy and $E_{17,11}$ is the sub-band energy.

Under the condition $\eta_g \to 0$ and $I \to 0$, Eqs. (1.152) and (1.155) assume the forms

$$\overline{G}_9 = I_{11}(E_{17,12}) \tag{1.154}$$

$$n_0 = \overline{G}_{10}F_{-1}(\eta_{1210}) \tag{1.155}$$

where

$\eta_{1210} = (k_BT)^{-1}(E_{F21} - E_{17,12})$, E_{F21} is the corresponding Fermi energy

and E_{F21} is the sub band energy.

(ii) From Eq. (1.87), the $E_{17,13}$ and n_0 become

$$\overline{G}_9 = T_{21}(E_{17.13}, \eta_g, \lambda) \tag{1.156}$$

$$n_0 = \overline{G}_{10}F_{-1}(\eta_{129}) \tag{1.157}$$

where

$$\eta_{129} = (k_BT)^{-1}(E_{F20} - E_{17,13})$$

Under the condition $\eta_g \to 0$, Eqs. (1.156) and (1.157) assume the forms

$$\overline{G}_9 = \tau_0(E_{17,14}, \lambda) \tag{1.158}$$

$$n_0 = \overline{G}_{10} F_{-1}(\eta_{1210}) \tag{1.159}$$

where

$\eta_{1210} = (k_B T)^{-1}(E_{F20} - E_{17,14})$ and $E_{17,14}$ is the sub-band energy.

Under the condition $\eta_g \to 0$ and $I \to 0$, Eqs. (1.158) and (1.159) assume the forms

$$\overline{G}_9 = E_{17,15}(1 + \alpha E_{17,15}) \tag{1.160}$$

$$n_0 = \overline{G}_{10} F_{-1}(\eta_{1211}) \tag{1.161}$$

where

$\eta_{1211} = (k_B T)^{-1}(E_{F20} - E_{17,15})$ and E_{F20} is the sub-band energy.

(iii) The sub-band energy ($E_{17,16}$) and n_0 can be expressed from Eq. (1.96) as

$$\overline{G}_9 = T_{31}(E_{17,16}, \eta_g, \lambda) \tag{1.162}$$

$$n_0 = \overline{G}_{10} F_{-1}(\eta_{1210}) \tag{1.163}$$

where

$$\eta_{1210} = (k_B T)^{-1}(E_{F21} - E_{17,16})$$

Under the condition $\eta_g \to 0$, Eqs. (1.156) and (1.157) assume the forms

$$\overline{G}_9 = \rho_0\big(E_{17,17}, \lambda\big) \tag{1.164}$$

$$n_0 = \overline{G}_{10} F_{-1}(\eta_{1211}) \tag{1.165}$$

where

$\eta_{1211} = (k_B T)^{-1}(E_{F21} - E_{17,17})$ and $E_{17.17}$ is the sub-band energy.

Under the condition $\eta_g \to 0$ and $I \to 0$, Eqs. (1.164) and (1.165) assume the forms

$$\overline{G}_9 = E_{17,18} \tag{1.166}$$

$$n_0 = \overline{G}_{10} F_{-1}(\eta_{1212}) \tag{1.167}$$

where

$\eta_{1212} = (k_B T)^{-1}(E_{F21} - E_{17,18})$ and $E_{17,18}$ is the sub-band energy.

1.2.8 The Nanowires

(a) The 1D DR can be written following (1.8) as

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = T_{11}(E, \eta_g, \lambda) \tag{1.168}$$

where, $\overline{G}_{11} = \frac{\hbar^2 (n_z \pi / d_z)^2}{2m_c} + \frac{\hbar^2 (n_y \pi / d_y)^2}{2m_c}$.

Using (1.168), the EEM is given by

$$m^*(E_{F22}, \eta_g, \lambda)/m_c = \text{R.P.}[T_{11}(E_{F22}, \eta_g, \lambda)]' \tag{1.169}$$

where E_{F22} is the corresponding Fermi energy.

Using Eq. (1.168), $C_{1D} = \pi$ and summing over both n_y and n_z the n_0 can be written as

$$n_{1D} = \overline{G}_{12}[\text{R.P.}[T_{3L1}(E_{F22}, n_y, n_z, \eta_g, \lambda)]] \tag{1.170}$$

where, $\overline{G}_{12} = (\frac{2g_v\sqrt{2m_c}}{\pi\hbar} \sum_{n_y=1}^{n_{y\max}} \sum_{n_z=1}^{n_{z\max}})$ and $T_{3L1}(E_{F22}, n_y, n_z, \eta_g, \lambda) = [[T_{11}(E_{F22}, \eta_g, \lambda) - [\overline{G}_{11}]]]^{\frac{1}{2}}$.

The 1D DR for NWs when $\eta_g \to 0$ can be written following (1.1d) as

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = \beta_0(E, \lambda) \tag{1.171}$$

Using Eq. (1.171), the EEM becomes

$$m^*(E_{F23}, \eta_g, \lambda)/m_c = [\beta_0(E_{F23}, \lambda)]' \tag{1.172}$$

where E_{F23} is the corresponding Fermi energy.

The ES is given by

$$n_{1D} = \overline{G}_{12}\left[\left[\beta_0(E_{F23}, \lambda) - [\overline{G}_{11}]\right]\frac{2m_c}{\hbar^2}\right]^{1/2} \tag{1.173}$$

(b) Using Eq. (1.9), the 1D DR becomes

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = T_{21}(E, \eta_g, \lambda) \tag{1.174}$$

Using (1.173), the EEM assumes the form

$$m^*(E_{F22}, \eta_g, \lambda)/m_c = [T'_{21}(E_{F22}, \eta_g, \lambda)] \tag{1.175}$$

The ES can be written as

$$n_{1D} = \overline{G}_{12}\left[\left[T_{21}(E_{F22}, \eta_g, \lambda) - [\overline{G}_{11}]\right]\frac{2m_c}{\hbar^2}\right]^{1/2} \tag{1.176}$$

Using Eq. (1.2), the 1D DR for NWs when $\eta_g \to 0$ becomes

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = \tau_0(E, \lambda) \tag{1.177}$$

Using Eq. (1.177), the EEM becomes

$$m^*(E_{F23}, \eta_g, \lambda)/m_c = [\tau_0(E_{F23}, \lambda)]' \tag{1.178}$$

where E_{F23} is the corresponding Fermi energy.

The ES assumes the form

$$n_{1D} = \overline{G}_{12}\left[\left[\tau_0(E_{F23}, \lambda) - [\overline{G}_{11}]\right]\frac{2m_c}{\hbar^2}\right]^{1/2} \tag{1.179}$$

(c) Using Eq. (1.10), the $1D$ DR becomes

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = T_{31}(E, \eta_g, \lambda) \tag{1.180}$$

Using Eq. (1.180), the EEM becomes

$$m^*(E_{F22}, \eta_g, \lambda)/m_c = [T'_{31}(E_{F22}, \eta_g, \lambda)] \tag{1.181}$$

The ES assumes the form

$$n_{1D} = \overline{G}_{12}\left[\left[T_{31}(E_{F22}, \eta_g, \lambda) - [\overline{G}_{11}]\right]\frac{2m_c}{\hbar^2}\right]^{1/2} \tag{1.182}$$

Using Eq. (1.3), the $1D$ DR for NWs when $\eta_g \to 0$ becomes

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = \rho_0(E, \lambda) \tag{1.183a}$$

Using Eq. (1.183a), the EEM becomes

$$m^*(E_{F23}, \eta_g, \lambda)/m_c = [\rho_0'(E_{F23}, \lambda)] \tag{1.183b}$$

The ES assumes the form

$$n_{1D} = \overline{G}_{12}\left[\left[\rho_0(E_{F23}, \lambda) - [\overline{G}_{11}]\right]\frac{2m_c}{\hbar^2}\right]^{1/2}. \tag{1.184}$$

1.2.9 The Magneto Doping Superlattices

(a) The magneto DR and ES in HDDSL can be written from (1.8) as

$$T_{11}(E_{24}, \eta_g, \lambda) = \overline{G}_{14} \tag{1.185}$$

and

$$n_0 = \overline{G}_{15}[\text{R.P. } F_{-1}(\eta_{17,40})] \tag{1.186}$$

where, E_{24} is the sub-band energy,

$$\overline{G}_{14} = \left(n_i + \frac{1}{2}\right)\hbar\omega_{91\text{HD}}(E_{24}, \eta_g, \lambda) + \left(n + \frac{1}{2}\right)\hbar\omega_0,$$

$$\overline{G}_{15} = \frac{g_v eB}{\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{i=0}^{i_{\max}}, \eta_{17,40} = (k_B T)^{-1}[E_{F24} - E_{24}].$$

and E_{F24} is the corresponding Fermi energy.

(b) Eqs. (1.185) and (1.186) for Eq. (1.9) assume the forms

$$T_{21}(E_{25}, \eta_g, \lambda) = \overline{G}_{16} \tag{1.187}$$

and

$$n_0 = \overline{G}_{15}F_{-1}(\eta_{17.41}) \tag{1.188}$$

where, E_{25} is the sub-band energy,

$$\overline{G}_{16} = \left(n_i + \frac{1}{2}\right)\hbar\omega_{92\text{HD}}(E_{25}, \eta_g, \lambda) + \left(n + \frac{1}{2}\right)\hbar\omega_0,$$

$$\eta_{17.41} = (k_B T)^{-1}[E_{F25} - E_{25}]$$

and E_{F25} is the corresponding Fermi energy.

(c) Eqs. (1.187) and (1.188) for Eq. (1.10) assume the forms

$$T_{31}(E_{26}, \eta_g, \lambda) = \overline{G}_{17} \tag{1.189}$$

and

$$n_0 = \overline{G}_{15} F_{-1}(\eta_{17.42}) \tag{1.190}$$

where, $\overline{G}_{17} = \left(n_i + \frac{1}{2}\right)\hbar\omega_{93\text{HD}}(E_{26}, \eta_g, \lambda) + \left(n + \frac{1}{2}\right)\hbar\omega_0$,

$\eta_{17.42} = (k_B T)^{-1}[E_{F26} - E_{26}]$

and E_{F26} is the corresponding Fermi energy.

1.2.10 The Quantum Well Effective Mass Superlattices

(a) Using Eq. (1.8), the DR becomes [24]

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD1}}(E, k_y, k_z\lambda))\}^{-2} - k_\perp^2\right] \tag{1.191}$$

Using Eq. (1.191), the DR in this case becomes

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD1}}(E, k_y, k_z, \lambda))\}^2 - k_\perp^2\right] \tag{1.192}$$

Using Eq. (1.192), the EEM and EnSLSHD1 become

$$m^*(k_\perp, E_{F27}, \lambda) = \frac{\hbar^2}{L_0^2}\text{R.P.}\left|\left[\frac{\cos^{-1}[f_{\text{HD1}}(E_{F27}, k_y, k_z) f_{\text{HD1}}(E_{F27}, k_y, k_z)}{\sqrt{1 - f_{\text{HD1}}^2(E_{F27}, k_y, k_z)}}\right]\right| \tag{1.193}$$

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD1}}(E_{\text{nSLSHD1}}, 0, 0, \lambda))\}^2\right] \tag{1.194}$$

(b) Using Eq. (1.9), the DR becomes [24]

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD2}}(E, k_y, k_z\lambda))\}^{-2} - k_\perp^2\right] \tag{1.195}$$

Using Eq. (1.195), the DR in this case becomes

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD2}}(E,k_y,k_z,\lambda))\}^2 - k_\perp^2\right] \tag{1.196}$$

Using Eq. (1.196), the EEM and EnSLSHD2 become

$$m^*(k_\perp,E,\lambda) = \frac{\hbar^2}{L_0^2}\left|\left[\frac{\cos^{-1}[f_{\text{HD2}}(E,k_y,k_z)f_{\text{HD2}}(E,k_y,k_z)}{\sqrt{1-f_{\text{HD2}}^2(E,k_y,k_z)}}\right]\right| \tag{1.197}$$

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD2}}(E_{\text{nSLSHD2}},0,0,\lambda))\}^2\right] \tag{1.198}$$

(c) Using Eq. (1.10), the DR becomes [24]

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD3}}(E,k_y,k_z\lambda))\}^{-2} - k_\perp^2\right] \tag{1.199}$$

Using (1.199), the DR in QWHD effective mass superlattices assumes the forms

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD3}}(E,k_y,k_z,\lambda))\}^2 - k_\perp^2\right] \tag{1.200}$$

Using Eq. (1.200), the EEM and EnSLSHD3 become

$$m^*(k_\perp,E,\lambda) = \frac{\hbar^2}{L_0^2}\left|\left[\frac{\cos^{-1}[f_{\text{HD3}}(E,k_y,k_z)f_{\text{HD3}}(E,k_y,k_z)}{\sqrt{1-f_{\text{HD3}}^2(E,k_y,k_z)}}\right]\right| \tag{1.201}$$

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD3}}(E_{\text{nSLSHD3}},0,0,\lambda))\}^2\right] \tag{1.202}$$

1.2.11 The Nanowire Effective Mass Superlattices

(a) The DR, the EEM, the sub-band energy (E_{nSL5HD4}) and the ES in NWHD effective mass superlattices on the basis of Eq. (1.8) assume the form

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD1}}(\overline{Z}_1,\lambda))\}^2 - [\overline{G}_{36}]\right] \tag{1.203}$$

$$m^*(\overline{Z}_1,\lambda)=\frac{\hbar^2}{L_0^2}\text{R.P.}\left|\left[\frac{\cos^{-1}[f_{\text{HD1}}(\overline{Z}_1)f_{\text{HD1}}(\overline{Z}_1)}{\sqrt{1-f_{\text{HD1}}^2(\overline{Z}_1)}}\right]\right| \tag{1.204}$$

$$0=\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\text{HD1}}\left(E_{\text{nSL5HD4}},\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z},\lambda\right)\right)\right\}^2-[\overline{G}_{36}]\right] \tag{1.205}$$

$$n_0=\frac{2}{\pi}\text{R.P.}\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\text{HD1}}\left(E_{F27},\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z},\lambda\right)\right)\right\}^2-[\bar{G}_{36}]\right]\right]^{1/2} \tag{1.206}$$

where $\overline{Z}_1=(E,\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z})$, $\overline{G}_{36}=[(\frac{n_y\pi}{d_y})^2+(\frac{n_z\pi}{d_z})^2]$.

(b) Equations (1.203), (1.204), (1.205) and (1.206) for Eq. (1.9) assume the forms

$$k_x^2=\left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD2}}(\overline{Z}_1,\lambda))\}^2-[\overline{G}_{36}]\right] \tag{1.207}$$

$$m^*(\overline{Z}_1,\lambda)=\frac{\hbar^2}{L_0^2}\left|\left[\frac{\cos^{-1}[f_{\text{HD2}}(\overline{Z}_1)f_{\text{HD2}}(\overline{Z}_1)}{\sqrt{1-f_{\text{HD2}}^2(\overline{Z}_1)}}\right]\right| \tag{1.208}$$

$$0=\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\text{HD2}}\left(E_{\text{nSL5HD5}},\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z},\lambda\right)\right)\right\}^2-[\overline{G}_{36}]\right] \tag{1.209}$$

$$n_0=\frac{2}{\pi}\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\text{HD2}}\left(E_{F28},\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z},\lambda\right)\right)\right\}^2-[\overline{G}_{36}]^{1/2}\right]\right] \tag{1.210}$$

(c) Equations (1.207), (1.208), (1.209) and (1.210) for Eq. (1.10) assume the forms

$$k_x^2=\left[\frac{1}{L_0^2}\{\cos^{-1}(f_{\text{HD3}}(\overline{Z}_1,\lambda))\}^2-[\overline{G}_{36}]\right] \tag{1.211}$$

$$m^*(\overline{Z}_1,\lambda)=\frac{\hbar^2}{L_0^2}\left|\left[\frac{\cos^{-1}[f_{\text{HD3}}(\overline{Z}_1)f_{\text{HD3}}(\overline{Z}_1)}{\sqrt{1-f_{\text{HD3}}^2(\overline{Z}_1)}}\right]\right| \tag{1.212}$$

$$0=\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\text{HD3}}\left(E_{\text{nSL5HD6}},\frac{n_y\pi}{d_y},\frac{n_z\pi}{d_z},\lambda\right)\right)\right\}^2-[\overline{G}_{36}]\right] \tag{1.213}$$

$$n_0 = \frac{2}{\pi}\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\mathrm{HD3}}\left(E_{F29}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_z}, \lambda\right)\right)\right\}^2 - [\overline{G}_{36}]\right]\right]^{1/2}. \tag{1.214}$$

1.2.12 The Quantum Dot Effective Mass Superlattices

(a) The DR and the ES in QDHD effective mass superlattices on the basis of Eq. (1.8) assume the form

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\mathrm{HD1}}\left(E_{30}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_z}, \lambda\right)\right)\right\}^2 - [\overline{G}_{36}]\right] \tag{1.215}$$

$$n_0 = [\overline{G}_{30}\mathrm{R.P.}\,[F_{-1}(\eta_{30})]] \tag{1.216}$$

where, E_{30} is the sub-band energy, $\overline{G}_{30} = \frac{2}{d_x d_y d_z}\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}, \eta_{30} = (k_B T)^{-1}[E_{F30} - E_{30}]$.

(b) Equations (1.215) and (1.216) for Eq. (1.9) assume the forms

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\mathrm{HD2}}\left(E_{31}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_z}, \lambda\right)\right)\right\}^2 - [\overline{G}_{36}]\right] \tag{1.217}$$

$$n_0 = [\overline{G}_{30}\,[F_{-1}(\eta_{31})]] \tag{1.218}$$

where, E_{31} is the sub-band energy, $\eta_{31} = (k_B T)^{-1}[E_{F31} - E_{31}]$ and E_{F31} is the corresponding Fermi energy.

(c) Equations (1.217) and (1.218) for Eq. (1.10) assume the forms

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{\mathrm{HD3}}\left(E_{32}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_z}, \lambda\right)\right)\right\}^2 - [\overline{G}_{36}]\right] \tag{1.219}$$

$$n_0 = [\overline{G}_{30}\,[F_{-1}(\eta_{32})]] \tag{1.220}$$

where, $\eta_{32} = (k_B T)^{-1}[E_{F32} - E_{32}]$.

1.2.13 The Magneto Effective Mass Superlattices

(a) Using Eq. (1.191), the DR can be expressed as [29]

$$k_x^2 = [\rho_{4\mathrm{HD1}}(n, E, \lambda)] \tag{1.221}$$

Using (1.221), we get

$$m^*(n, E_{F33}, \lambda) = \mathrm{R.P.}\frac{\hbar^2}{2}[\rho_{4\mathrm{HD}}(n, E_{F33}, \lambda)]' \tag{1.222}$$

where, E_{F33} is the corresponding Fermi energy.

The n_0 versus E_{F33} relation assumes the form

$$n_0 = \overline{G}_{34}[\mathrm{R.P.}[[\rho_{4\mathrm{HD1}}(n, E_{F33}, \lambda)]^{1/2}]] \tag{1.223}$$

where, $\overline{G}_{34} = \frac{eB}{\pi^2\hbar}\sum_{n=0}^{n_{\max}}$.

(b) For Eq. (1.9), the magneto DR assumes the form

$$k_x^2 = [\rho_{4\mathrm{HD2}}(n, E, \lambda)] \tag{1.224}$$

in which,

$$\rho_{4\mathrm{HD2}}(n, E, \lambda) = \frac{1}{L_0^2}[\cos^{-1}(f_{\mathrm{HD2}}(n, E, \lambda))]^2 - \left\{\frac{2|e|B}{\hbar}\left(n + \frac{1}{2}\right)\right\}$$

$$f_{\mathrm{HD2}}(E, n, \lambda) = a_{1\mathrm{HD2}}\cos[a_0 C_{1\mathrm{HD2}}(E, n, \lambda) + b_0 D_{1\mathrm{HD2}}(E, n, \lambda)] - a_{2\mathrm{HD2}}\cos[a_0 C_{1\mathrm{HD2}}(E, n, \lambda) - b_0 D_{1\mathrm{HD2}}(E, n, \lambda)] - \circ$$

$$C_{1\mathrm{HD2}}(E, n, \lambda) \equiv \left[\left(\frac{2m_{c1}}{\hbar^2}\right)T_2(E, \eta_{g_1}, \lambda) - \left\{\frac{2|e|B}{\hbar}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}$$

and

$$D_{1\mathrm{HD2}}(E, n, \lambda) \equiv \left[\left(\frac{2m_{c2}}{\hbar^2}\right)T_2(E, \eta_{g_2}, \lambda) - \left\{\frac{2|e|B}{\hbar}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}$$

The EEM and the ES assume the forms

$$m^*(n, E_{F34}, \lambda) = \frac{\hbar^2}{2}[\rho_{4\mathrm{HD2}}(n, E_{F34}, \lambda)]' \tag{1.225}$$

$$n_0 = [\overline{G}_{34}[[\rho_{4\mathrm{HD2}}(n, E_{F34}, \lambda)]^{1/2}]] \tag{1.226}$$

where E_{F34} is the corresponding Fermi energy.

(c) For Eq. (1.10), the magneto DR assumes the form

$$k_x^2 = [\rho_{4\text{HD3}}(n, E, \lambda)] \tag{1.227}$$

The EEM and ES assume the forms

$$m^*(n, E_{F35}, \lambda) = \frac{\hbar^2}{2}[\rho_{4\text{HD3}}(n, E_{F35}, \lambda)]' \tag{1.228}$$

$$n_0 = [\overline{G}_{34}[[\rho_{4\text{HD3}}(n, E_{F35}, \lambda)]^{1/2}]] \tag{1.229}$$

where E_{F35} is the corresponding Fermi energy.

1.2.14 The Magneto Quantum Well Effective Mass Superlattices

Using (1.221), the DR and n_0 assumes the form

$$\left(\frac{n_x\pi}{d_x}\right)^2 = [\rho_{4\text{HGD1}}(n, E_{36}, \lambda)] \tag{1.230}$$

$$n_0 = [\overline{G}_{35}\text{R.P. } F_{-1}(\eta_{36})] \tag{1.231}$$

where, E_{36} is the sub-band energy, $\bar{G}_{35} = \frac{g_v eB}{\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{n_x=0}^{n_{x\max}}$, $\eta_{36} = (k_BT)^{-1}[E_{F36} - E_{36}]$.

Equations (1.230) and (1.231) for Eq. (1.9) get transformed as

$$\left(\frac{n_x\pi}{d_x}\right)^2 = [\rho_{4\text{HGD2}}(n, E_{37}, \lambda)] \tag{1.232}$$

$$n_0 = [\overline{G}_{35}F_{-1}(\eta_{37})] \tag{1.233}$$

where, E_{37} is the sub-band energy, $\eta_{37} = (k_BT)^{-1}[E_{F37} - E_{37}]$.

and E_{F37} is the corresponding Fermi energy.

(c) Equations (1.232) and (1.233) for Eq. (1.10) get transformed as

$$\left(\frac{n_x\pi}{d_x}\right)^2 = [\rho_{4\text{HGD3}}(n, E_{38}, \lambda)] \tag{1.234}$$

$$n_0 = [\overline{G}_{35} F_{-1}(\eta_{38})] \tag{1.235}$$

where, $\eta_{38} = (k_B T)^{-1}[E_{F38} - E_{38}]$ and E_{F38} is the corresponding Fermi energy.

1.2.15 The Quantum Well Superlattices with Graded Interfaces

Using Eq. (1.10), the DR becomes [29]

$$\frac{\hbar^2 k^2}{2m_{cj}} = V_{1j}(E, \eta_{gj}, \lambda, \Delta_j, E_{g0j}) + i V_{2j}(E, \eta_{gj}, \lambda, \Delta_j, E_{g0j}) \tag{1.236}$$

where $j = 1, 2,\ V_{1j}(E, \eta_{gj}, \lambda, \Delta_j, E_{g0j}) = [U_{\lambda j} T_{1j}(E, \Delta_j, E_{gj}, \eta_{gj}) - P_{\lambda j}]$ and the other notations are defined in [24].

Therefore, the DR in HD III-V SLs with graded interfaces (GI) for $I \neq 0$assumes the form [29]

$$k_z^2 = G_8 + i H_8 \tag{1.237}$$

where $G_8 = [\frac{C_7^2 - D_7^2}{L_0^2} - k_s^2]$, $C_7 = \cos^{-1}(\overline{\omega}_7)$, $\overline{\omega}_7 = (2)^{\frac{-1}{2}}[(1 - G_7^2 - H_7^2) - \sqrt{(1 - G_7^2 - H_7^2)^2 + 4G_7^2}]^{\frac{1}{2}}$

and the other notations are defined in [29].

Following (1.237), the DR can be written as

$$\left(\frac{n_z \pi}{d_z}\right)^2 = G_8 + i H_8 \tag{1.238}$$

The sub-band energy (E_{39}) becomes

$$\left(\frac{n_z \pi}{d_z}\right)^2 = |G_8 + i H_8|_{k_s = 0 \text{ and } E = E_{39}} \tag{1.239}$$

The analytical evaluation of ES is not possible and numerical methods must be used for further study.

1.2.16 The Nanowire Superlattices with Graded Interfaces

The DR becomes [29]

$$k_z^2 = G_{8,17,50} + i\,H_{8,17,50} \tag{1.240}$$

The EEM and ES assume the forms

$$m^*(E, n_x, n_y, \lambda, \eta_g) = \frac{\hbar^2}{2}(G'_{8,17,50}) \tag{1.241}$$

and

$$n_0 = \frac{2g_v}{\pi} R.P. \sum_{n_x=1}^{n_{x\max}} \sum_{n_y=1}^{n_{y\max}} [[G_{8,17,50} + i\,H_{8,17,50}]\big|_{E=E_{F40}}]^{1/2}] \tag{1.242}$$

where E_{F40} is the corresponding Fermi energy.

1.2.17 The Quantum dot Superlattices with Graded Interfaces

From Eq. (1.140), the DR becomes

$$\left(\frac{n_z\pi}{d_z}\right)^2 = [G_{8,17,50} + i\,H_{8,17,50}]\big|_{E=E_{41}} \tag{1.243}$$

The ES becomes

$$n_0 = [\overline{G}_{30}\text{R.P. } F_{-1}(\eta_{41})] \tag{1.244}$$

where, $\eta_{41} = (k_B T)^{-1}[E_{F41} - E_{41}]$ and E_{F41} is the corresponding Fermi energy.

1.2.18 The Magneto Superlattices with Graded Interfaces

The magneto HDDR becomes [30]

$$k_z^2 = G_{8,17,54} + i\,H_{8,17,54} \tag{1.245}$$

The EEM and ES become

$$m^*(E,\lambda,\eta_g) = \frac{\hbar^2}{2}\left(G'_{8,17,54}\right) \tag{1.246}$$

$$n_0 = \frac{g_v eB}{\pi^2\hbar}\text{R.P.}\sum_{n=0}^{n_{\max}}[[G_{8,17,54} + iH_{8,17,54}]\big|_{E=E_{F42}}]^{1/2}]. \tag{1.247}$$

1.2.19 The Magneto Quantum Well Superlattices with Graded Interfaces

The magneto DR in QWHD superlattices becomes

$$\left(\frac{n_z\pi}{d_z}\right)^2 = [G_{8,17,54} + iH_{8,17,54}]\big|_{E=E_{43}} \tag{1.248}$$

The ES becomes

$$n_0 = [\overline{G}_{10}\text{R.P.}F_{-1}(\eta_{43})] \tag{1.249}$$

$\eta_{43} = (k_BT)^{-1}[E_{F43} - E_{43}]$ and E_{F43} is the corresponding Fermi energy.

1.3 Few Related Applications

1.3.1 Introduction

We shall now discuss a few ES-dependent electronic properties which find extensive applications [28, 33–179] in modern nano-devices.

1.3.2 Thermoelectric Power (G)

After the discovery of Quantum Hall Effect (QHE) [33], the thermoelectric powers under strong magnetic field (G) are being investigated under different physical conditions [34–63].

When $f(E) = 1$ G becomes [63]

$$|G| = \left(\frac{\pi^2 k_B^2 T}{3|e|n_0}\right)\left(\frac{\partial n_0}{\partial E_F}\right) \tag{1.250}$$

For 2*D* systems, Eq. (1.250) under electric quantum limit becomes

$$|G| = \frac{\pi^2 k_B^2 T}{3en_{02D}}\left[\frac{\partial n_{02D}}{\partial(E_{F2D} - E_{02D})}\right] \tag{1.251}$$

Under the condition of heavy doping, Eq. (1.250) gets transformed as

$$|G| = \frac{\pi^2 k_B^2 T}{3en_{0\mathrm{HD}}}\left[\frac{\partial n_{0\mathrm{HD}}}{\partial(E_{\mathrm{FHD}} - E_{0\mathrm{HD}})}\right] \tag{1.252}$$

Thus, we can study G by using the different expressions of ES in various cases.

1.3.3 Experimental Determination of Debye Screening Length (DSL)

The DSL has extensively been studied after its inception [64–76]

The 3*D* DSL becomes [66]

$$L_{3D} = \left[\frac{e^2}{\varepsilon_{sc}} \cdot \frac{\partial n_0}{\partial E_F}\right]^{-1/2} \tag{1.253}$$

Thus, we can write

$$L_{3D} = \left[\frac{\pi^2 k_B^2 \varepsilon_{sc}}{3e^3 n_0 G}\right]^{1/2} \tag{1.254}$$

Similarly, we can prove that L_{2D} for inversion layers, quantum wells and the doping superlattices in the quantum limit case assumes the form

$$L_{2D} = \frac{2\pi^2 k_B^2 T \varepsilon_{sc}}{3e^3 n_{2D} G} \tag{1.255}$$

Since with increasing n_0, $(n_0 G)^{-1}$ decreases, we can infer that both L_{3D} and L_{2D} increase with decreasing n_0.

1.3.4 Experimental Determination of Diffusivity-Mobility Ratio

The importance of the diffusivity-mobility ratio is well known since its inception in 1905 (the Year of Physics) by Einstein [64] and extensively been investigated [77–109].

For Bulk materials, the diffusivity-mobility ratio assumes the form

$$\frac{D}{\mu} = \frac{n_{0\mathrm{HD}}}{e}\left[\frac{\partial n_{0\mathrm{HD}}}{\partial(E_{\mathrm{FHD}} - E_{0\mathrm{HD}})}\right]^{-1} \tag{1.256}$$

For 2D systems, the DMR (under quantum limit) becomes [97]

$$\frac{D}{\mu} = \frac{n_{02D}}{e}\left[\frac{\partial n_{02D}}{\partial(E_{F2D} - E_{02D})}\right]^{-1} \tag{1.257}$$

Using (1.252), (1.256) and (1.257), one obtains

$$\frac{D}{\mu} = \left(\frac{\pi^2 k_B^2 T}{3|e|^2|G|}\right) \tag{1.258}$$

Since $|G|$ decreases with increasing n_0, we can infer that $\frac{D}{\mu}$ increases with increasing n_0.

1.3.5 Experimental Determination of ΔC_{44} and ΔC_{456}

The ΔC_{44} and ΔC_{456} have extensively been studied after its inception and can be written as [28, 110–176]

$$\Delta C_{44} = \frac{-G_0^2}{9}\left[\frac{\partial n_0}{\partial E_F}\right] \tag{1.259}$$

and

$$\Delta C_{456} = \frac{G_0^3}{27}\left[\frac{\partial^2 n_0}{\partial E_F^2}\right] \tag{1.260}$$

Using the appropriate equations, we get

$$\Delta C_{44} = [30k_B^2 T]^{-1} G[-n_0(G_0)^2|e|] \tag{1.261}$$

$$\Delta C_{456} = [300k_B^3 T]^{-1}\left[1 + \frac{n_0}{G}\frac{\partial G}{\partial n_0}\right]\left[n_0|e|(G_0)^3 G^2\right]. \tag{1.262}$$

1.3.6 Quantum Capacitance

With the advent of quantum MOSFET, the quantum capacitance C_g has extensively been studied in [180–183] and can be written as

$$C_{\mathrm{g}} = \frac{e\partial n_{\mathrm{s}}}{\partial V_{\mathrm{g}}} \tag{1.263}$$

where n_{s} is the surface ES per unit area and V_{g} is the gate voltage. Thus, we can study the C_{g} by using the ES in various cases.

1.4 Result and Discussions

Using the appropriate equations and parameters [184–228], we have plotted the normalized Fermi energy (Ω_1) under various conditions versus different physical variables for the indicated materials in the graphs as shown in Figs. 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.12, 1.13, 1.14, 1.15, 1.16, 1.17 and 1.18 to exhibit single ES-dependent electronic property, and we have further plotted the normalized

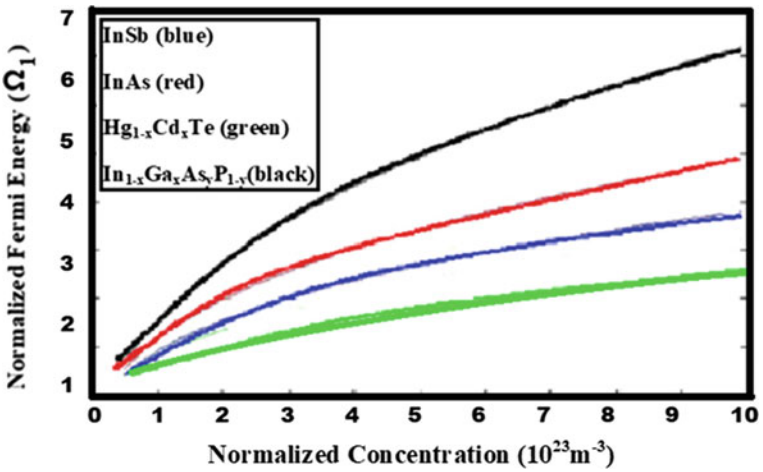

Fig. 1.1 Ω_1 versus n_0 for $I \neq 0$ for bulk indicated materials

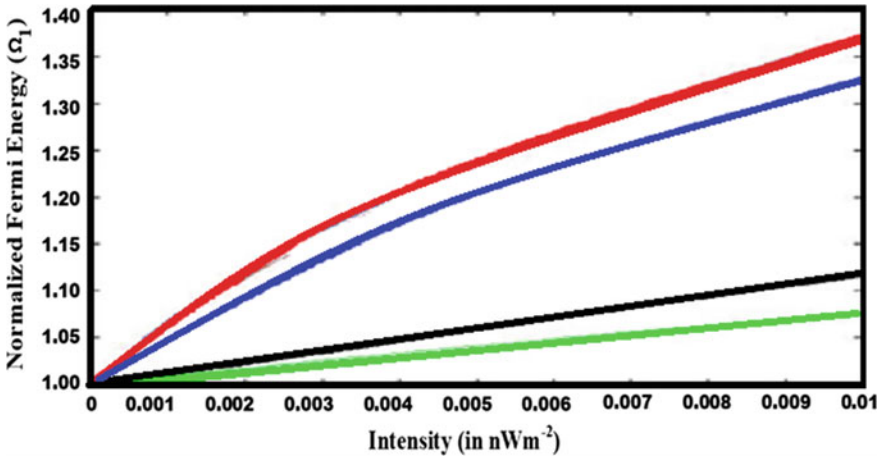

Fig. 1.2 Ω_1 versus I for all curves of Fig. 1.1

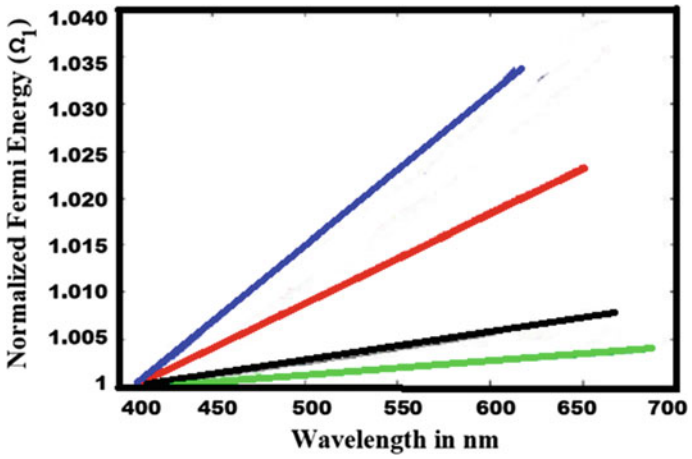

Fig. 1.3 Ω_1 versus λ for all curves of Fig. 1.1

EEM (Ω_2) versus various physical variables in Figs. 1.19, 1.20, 1.21, 1.22, 1.23, 1.24, 1.25, 1.26, 1.27, 1.28, 1.29, 1.30, 1.31, 1.32, 1.33, 1.34, 1.35, 1.36, 1.37, 1.38, 1.39, 1.40, 1.41, 1.42, 1.43, 1.44, 1.45, 1.46, 1.47, 1.48 and 1.49.

From the said figures, we note the following features:

1. From Figs. 1.1, 1.2, 1.3 and 1.4, we note that Ω_1 for bulk specimens of the indicated materials increases with increasing n_0, I, λ and decreasing x, respectively.
2. In Fig. 1.5, we note that magneto Ω_1 for $I \neq 0$ oscillates with $(1/B)$, for all the materials due to SdH effect.

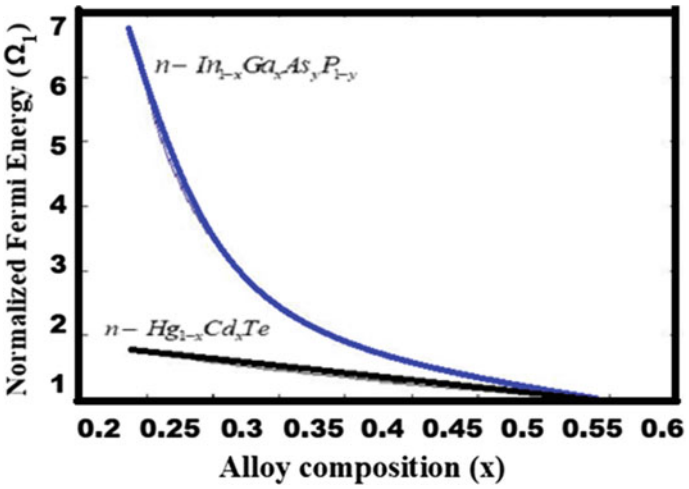

Fig. 1.4 Ω_1 versus x for the indicated opto-electronic materials

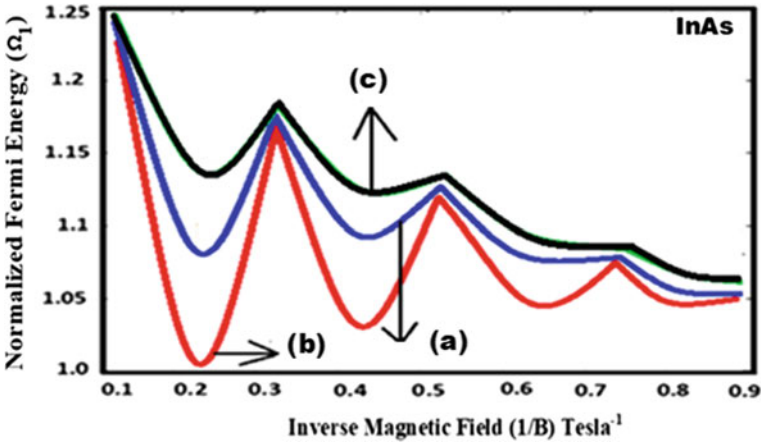

Fig. 1.5 Ω_1 versus $(1/B)$ for three curves of Fig. 1.1

3. From Figs. 1.6, 1.7 and 1.8, we note that magneto Ω_1 increases with increasing λ and I although rate of increment is different for different variables and decreases with increasing x.
4. From Fig. 1.9, we note that the magneto Ω_1 for $I \neq 0$ oscillates with n_0, for all the materials due to SdH effect.
5. From Fig. 1.10, we note that Ω_1 in QWs decreases with increasing d_y in oscillatory ways.
6. From Fig. 1.11, we note that Ω_1 in QWs increases with increasing n_0 in oscillatory ways.

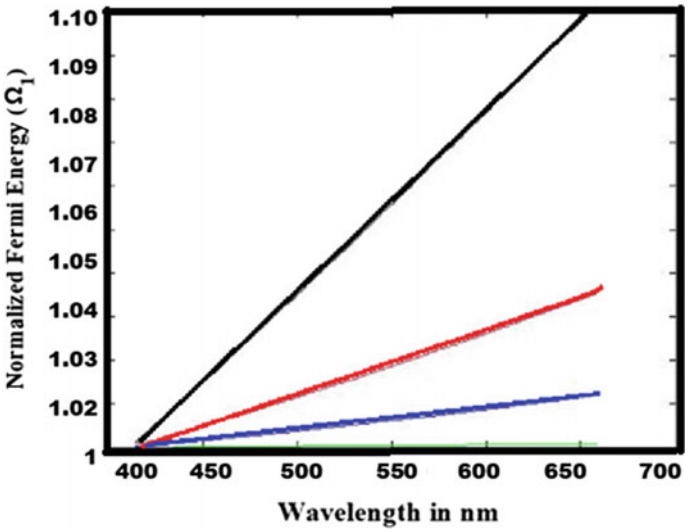

Fig. 1.6 Ω_1 versus λ when $B \neq 0$ and $I \neq 0$ for all curves of Fig. 1.1

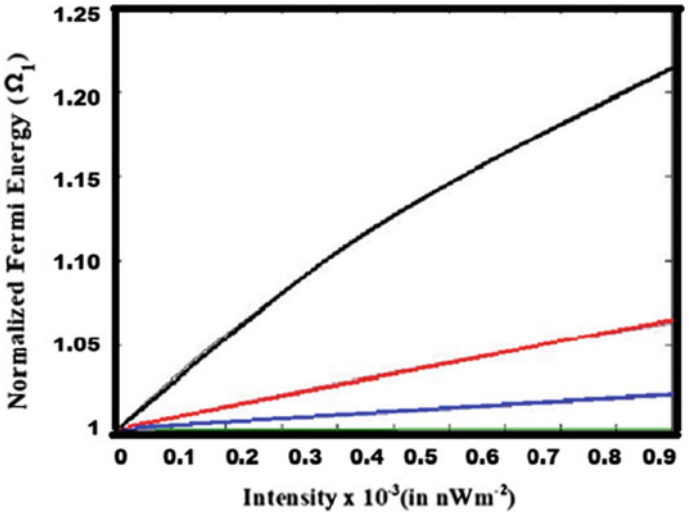

Fig. 1.7 Ω_1 versus I when $B \neq 0$ for all curves of Fig. 1.1

7. From Figs. 1.12, 1.13 and 1.14, we note that Ω_1 in QWs increases with increasing I and λ and decreases with increasing x, respectively.
8. From Figs. 1.15 and 1.16, we note that Ω_1 in NWs increases with decreasing d_z and increasing n_0 in oscillatory ways.

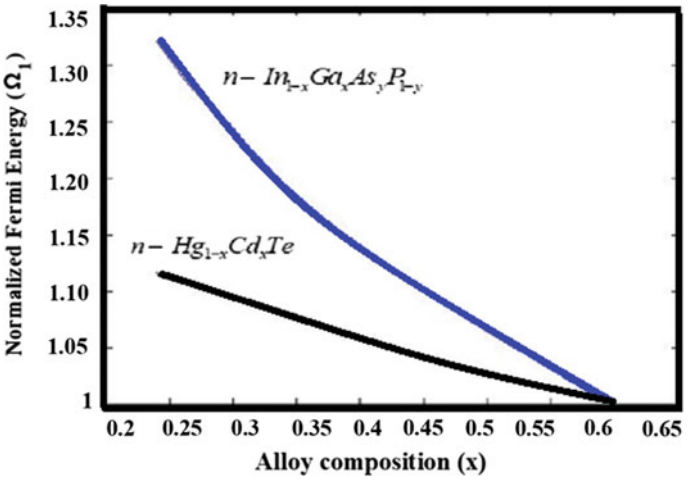

Fig. 1.8 Ω_1 versus x when $B \neq 0$ and $I \neq 0$ for the indicated materials

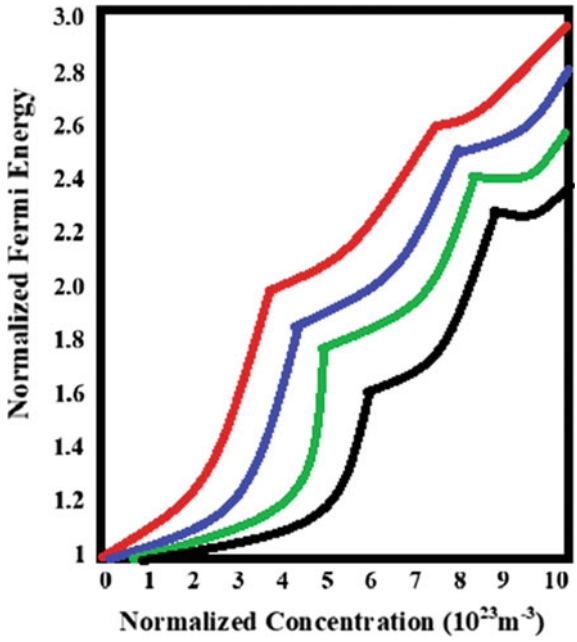

Fig. 1.9 Ω_1 versus n_0 when $B \neq 0$ and $I \neq 0$ for all curves of Fig. 1.1

9. From Figs. 1.17 and 1.18, we note that Ω_1 in NWs increases with increasing λ and decreasing x.
10. In Figs. 1.19, 1.20, 1.21 and 1.22, the normalized EEM (Ω_2) increases with increasing n_0 for all the indicated materials for both $I \neq 0$ and $I = 0$ whereas for parabolic energy bands Ω_2 is invariant of I, respectively.

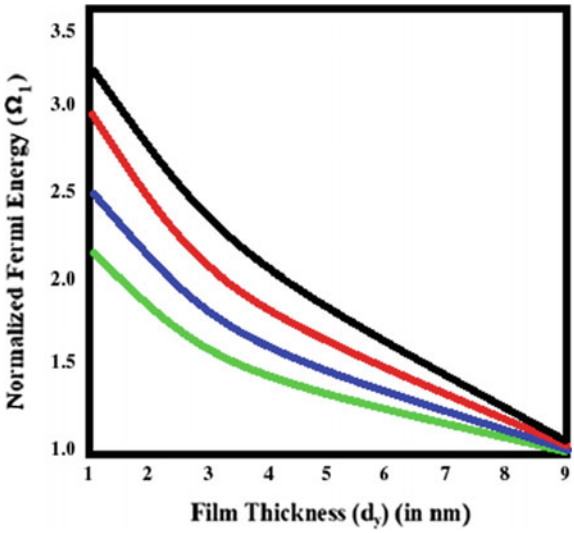

Fig. 1.10 Ω_1 versus d_y for QWs when $I \neq 0$ for all the curves of Fig. 1.1

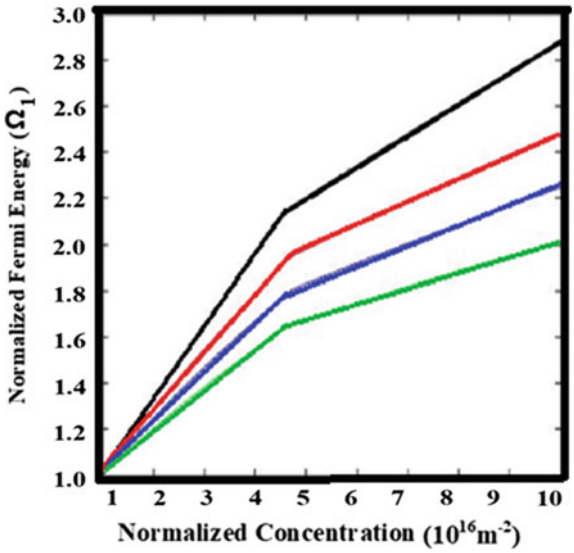

Fig. 1.11 Ω_1 versus n_0 for QWs when $I \neq 0$ for Fig. 1.1

11. From Figs. 1.23, 1.24, 1.25, 1.26, 1.27, 1.28, 1.29 and 1.30, we note that Ω_2 increases with increasing I and λ.
12. From Figs. 1.31 and 1.32, we note that Ω_2 changes with changing alloy composition.
13. From Fig. 1.33, we note that Ω_2 oscillates with $(1/B)$ due to SdH effect.
14. From Fig. 1.34, we note that Ω_2 oscillates with n_0 due to SdH effect.

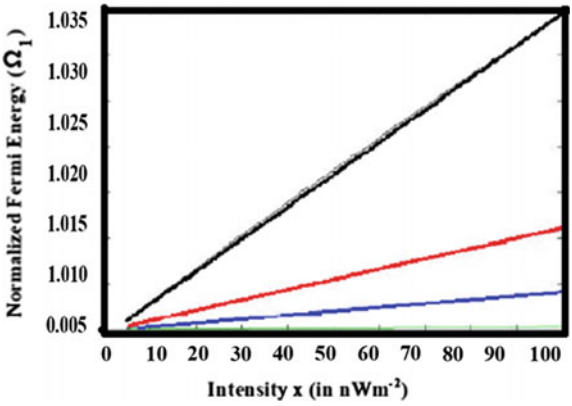

Fig. 1.12 Ω_1 versus I for QWs for Fig. 1.1

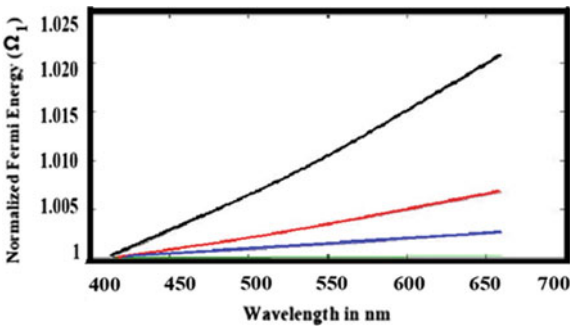

Fig. 1.13 Ω_1 versus λ for QWs for Fig. 1.1

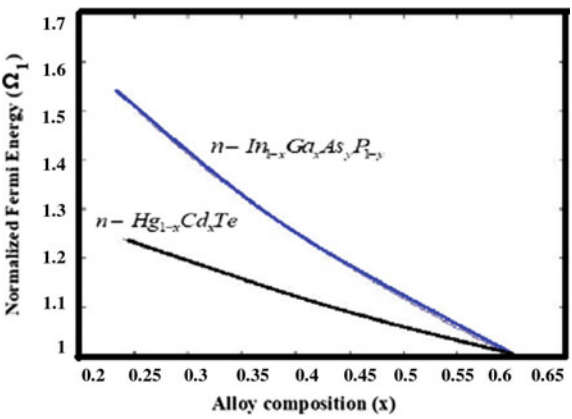

Fig. 1.14 Ω_1 versus x for QWs when $I \neq 0$ for the indicated materials

15. From Figs. 1.35 and 1.36, we note that Ω_2 increases with increasing I and λ in different ways.

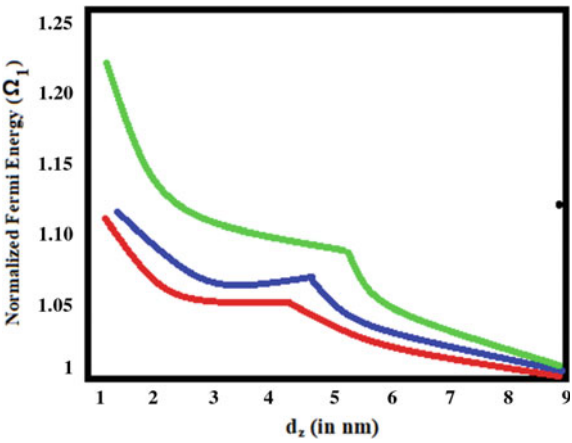

Fig. 1.15 Ω_1 versus d_z for NWs when $I \neq 0$ for Fig. 1.1

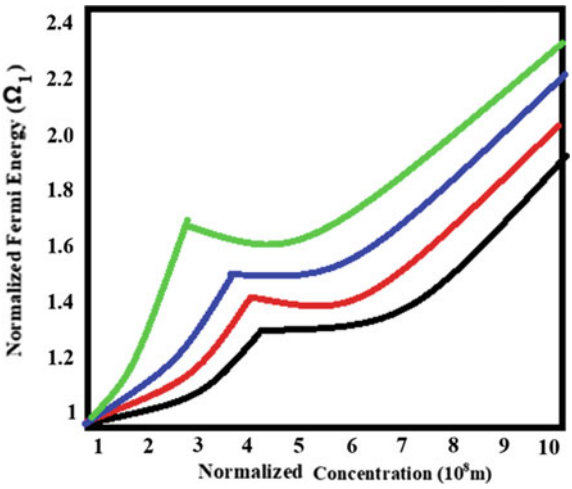

Fig. 1.16 Ω_1 versus n_0 for NWs when $I \neq 0$ for Fig. 1.1

16. From Fig. 1.37, we note that in QWs Ω_2 decreases with increasing d_z in perfectly periodic manner.
17. From Fig. 1.38, we observe that Ω_2 in QWs increases with increasing n_{2D}.
18. From Figs. 1.39 and 1.40, we note that in QWs Ω_2 increases with increasing I and λ in different ways.
19. The Ω_2 increases with increasing n_0 and λ for doping superlattices as shown in Figs. 1.41 and 1.42.
20. From Fig. 1.43, we observe that in NWs Ω_2 decreases with increasing d_y in perfectly periodic manner.
21. From Fig. 1.44, we note that in NWs Ω_2 increases with increasing n_{1D}.
22. In Fig. 1.45, Ω_2 oscillates with $(1/B)$ for $n = 0$ and $I \neq 0$for GaAs/AlGaAs HDEMSL for the indicated band models.

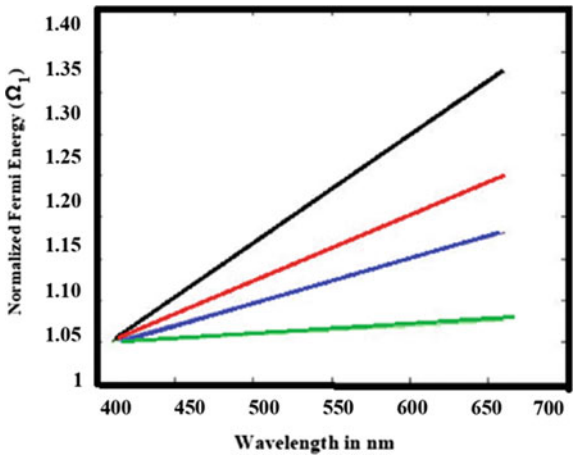

Fig. 1.17 Ω_1 versus λ for NWs for Fig. 1.1

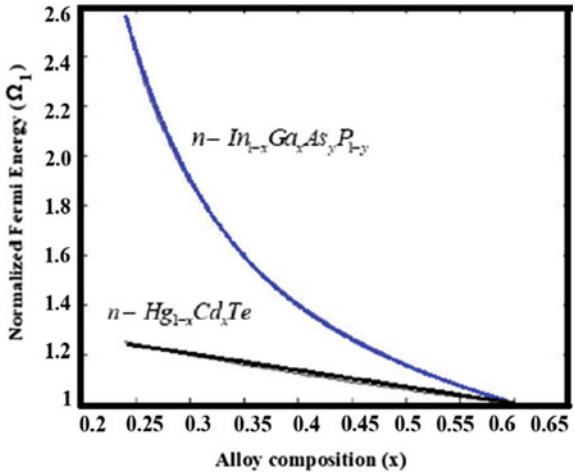

Fig. 1.18 Ω_1 versus x for NWs when $I \neq 0$ for the indicated materials

23. From Fig. 1.46, we note that Ω_2 increases with increasing n_0 for $n = 0$ and $I \neq 0$for GaAs/AlGaAs HDEMSL for the indicated band models.
24. From Fig. 1.47, we note that Ω_2 decreases with increasing d_z for both the quantum limits and $I \neq 0$for GaAs/AlGaAs HDNWEMSL for the indicated band models.
25. From Fig. 1.48, we note that Ω_2 increases with increasing I for both the quantum limits and $I \neq 0$for GaAs/AlGaAs HDNWEMSL for the indicated band models.
26. The last Fig. 1.49 indicates that Ω_2 increases with increasing n_{1D} for both the quantum limits and $I \neq 0$for GaAs/AlGaAs HDNWSL with graded interfaces for two-band Kane model.

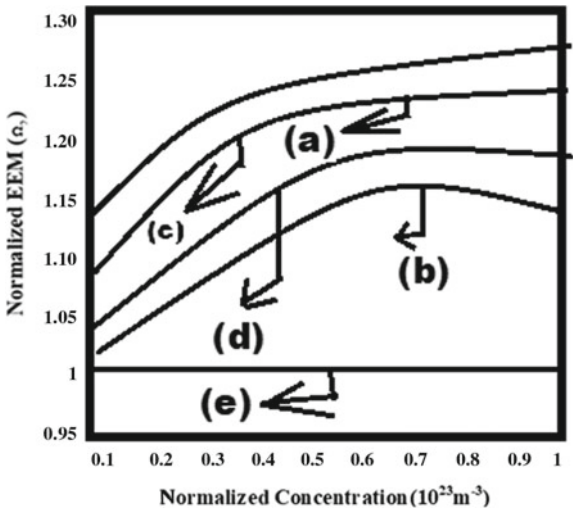

Fig. 1.19 Ω_2 versus n_0 for the indicated material for $I \neq 0$ where the curves **a** and **c** represent the three- and two-band models of Kane, respectively, and the curves **b** and **d** are for $I = 0$. The curve **e** represents the parabolic energy band model both in the presence and in the absence of I

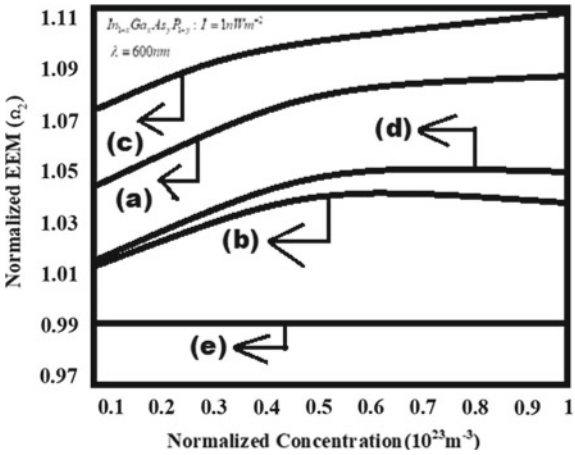

Fig. 1.20 Ω_2 versus n_0 for the indicated material for Fig. 1.19

Already 48 graphs for the ES and EEM versus different physical variables under strong radiation have been drawn, and to complete all the cases, the total number of graphs will exceed more than 100 for the first chapter only, and for the purpose of condensed presentation, we have not drawn these figures with the hope that our creative readers will perform the intricate computer programming for all the quantized materials as well under different physical conditions for the purpose of creating new physics for ES and other electronic properties as discussed here.

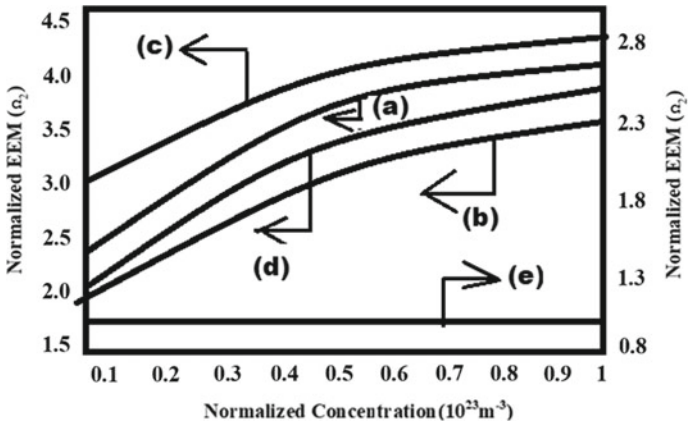

Fig. 1.21 Ω_2 versus n_0 for the indicated material for Fig. 1.19

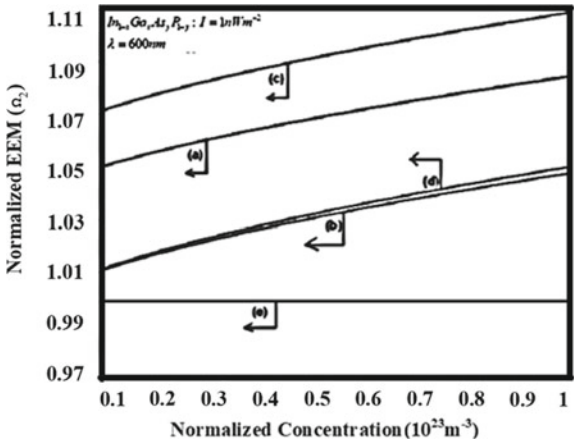

Fig. 1.22 Ω_2 versus n_0 for the indicated material for Fig. 1.19

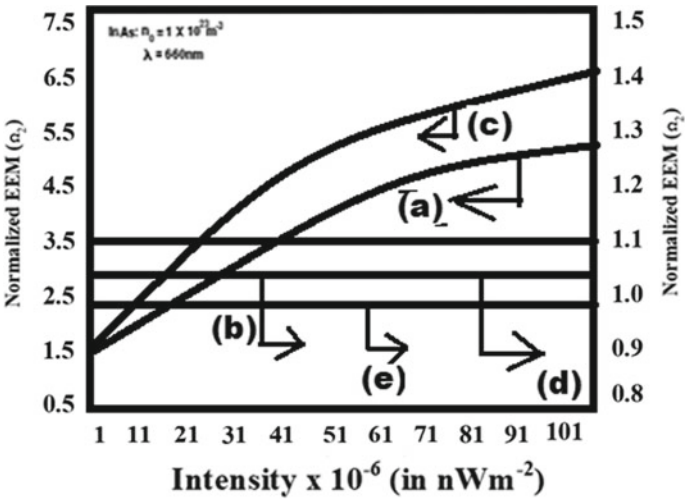

Fig. 1.23 Ω_2 versus I for the indicated material for Fig. 1.19

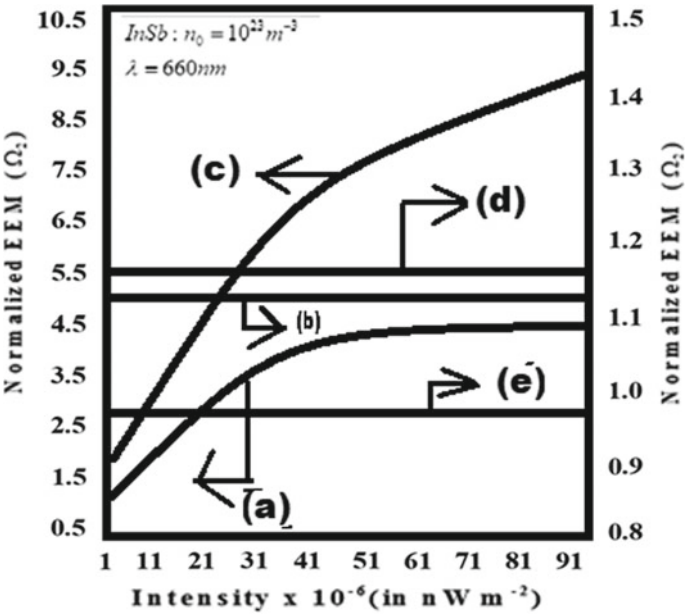

Fig. 1.24 Ω_2 versus I for the indicated material for Fig. 1.19

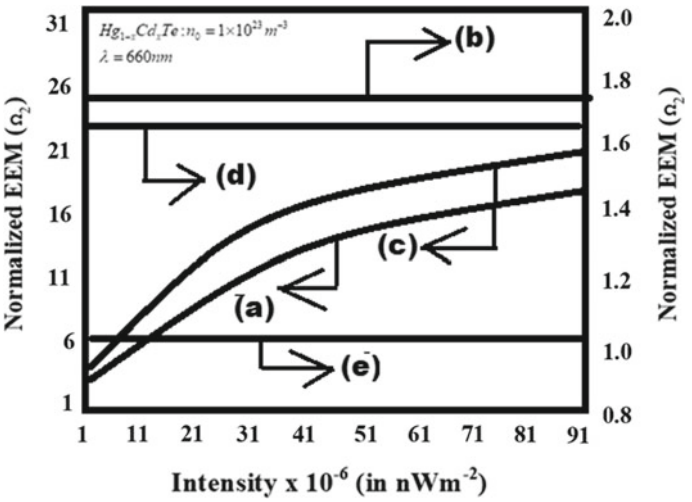

Fig. 1.25 Ω_2 versus I for the indicated material for Fig. 1.19

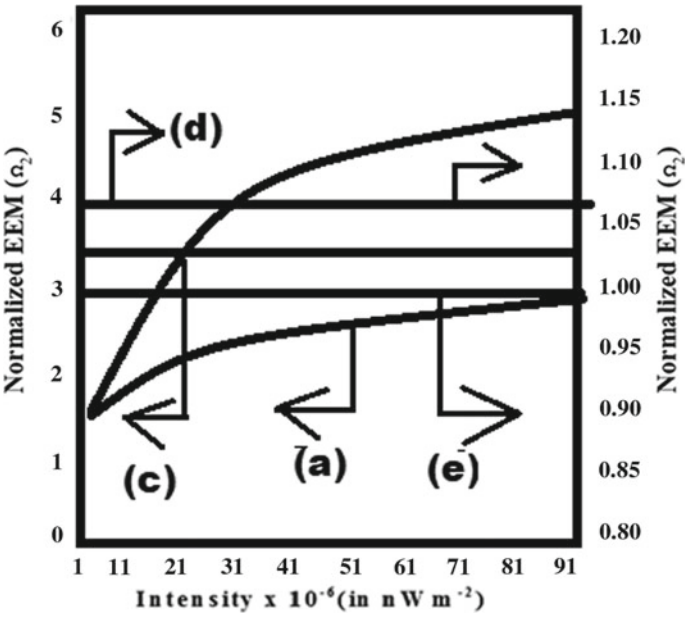

Fig. 1.26 Ω_2 versus I for the indicated material for Fig. 1.19

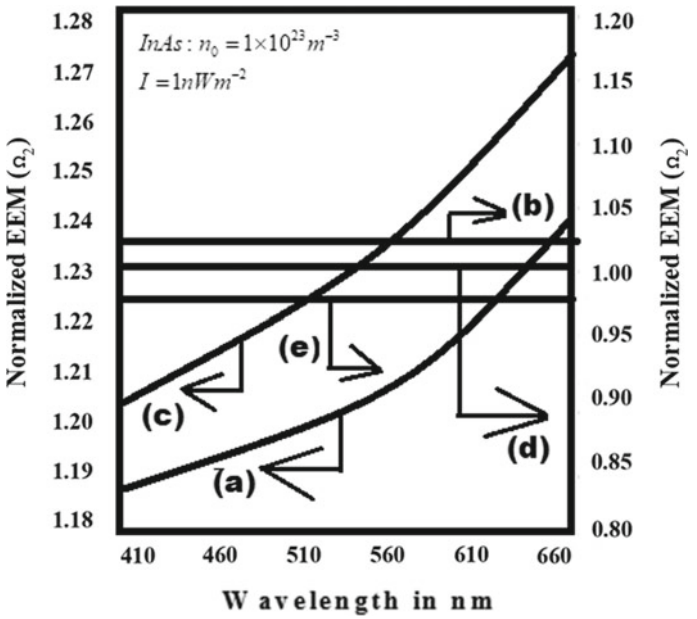

Fig. 1.27 Ω_2 versus λ for the indicated material for Fig. 1.19

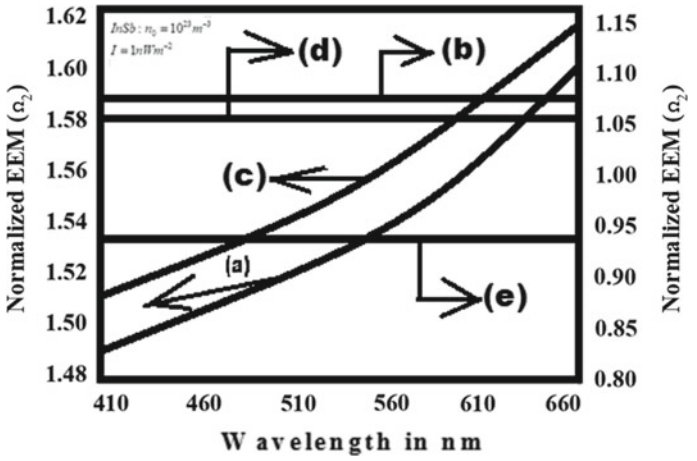

Fig. 1.28 Ω_2 versus λ for the indicated material for Fig. 1.19

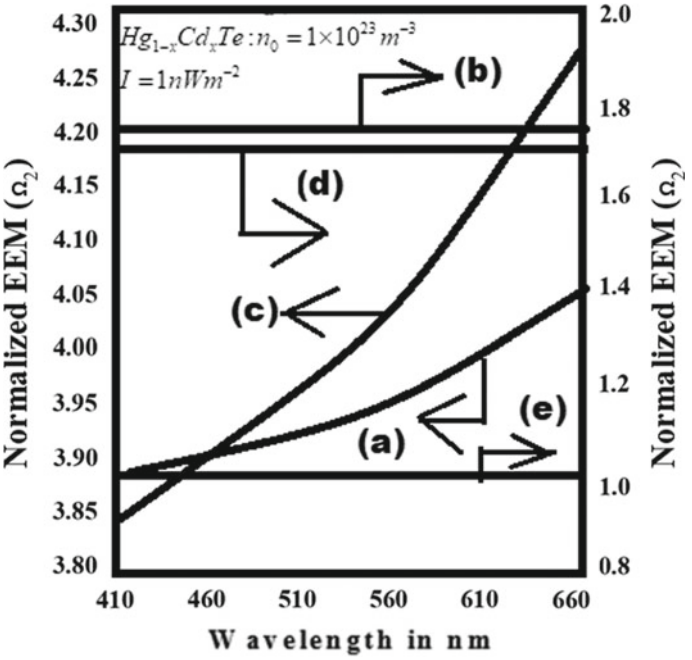

Fig. 1.29 Ω_2 versus λ for the indicated material for Fig. 1.19

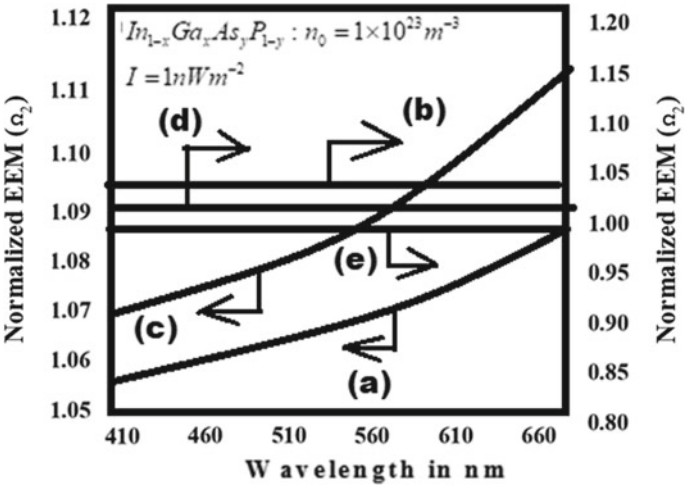

Fig. 1.30 Ω_2 versus λ for the indicated material for Fig. 1.19

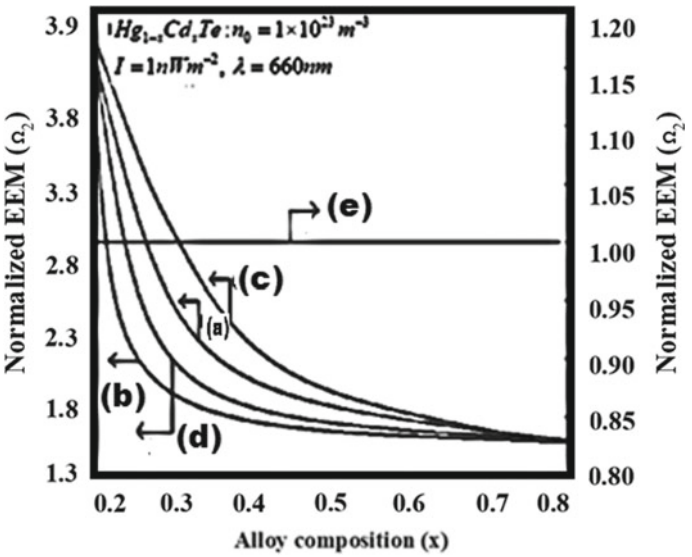

Fig. 1.31 Ω_2 versus x for the indicated material for Fig. 1.19

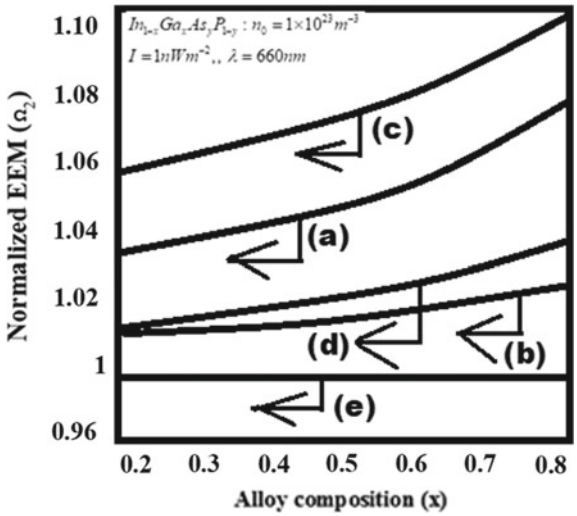

Fig. 1.32 Ω_2 versus y for the indicated material for Fig. 1.19

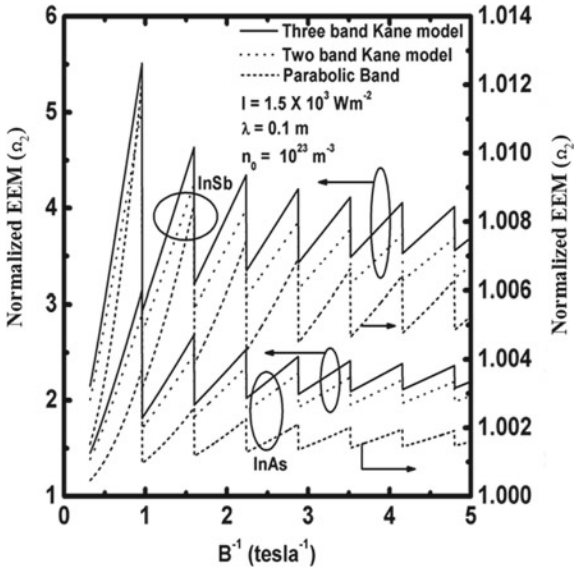

Fig. 1.33 Ω_2 versus $(1/B)$ for the indicated materials when $I \neq 0$

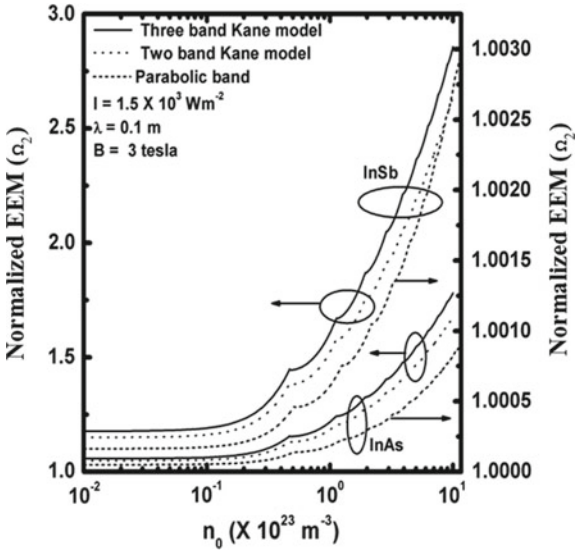

Fig. 1.34 Ω_2 versus n_0 for the indicated materials when $I \neq 0$

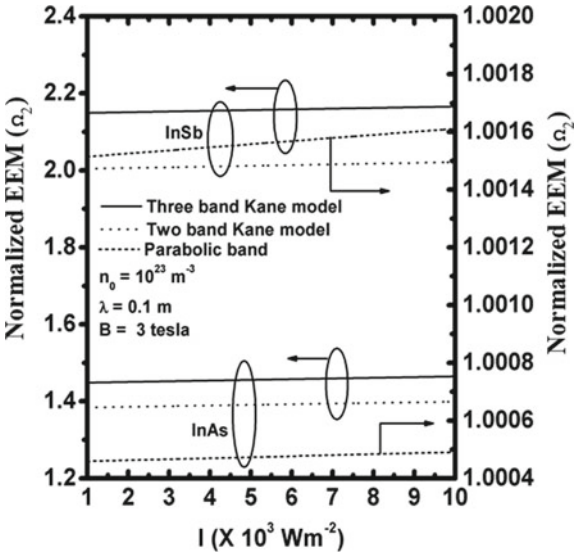

Fig. 1.35 Ω_2 versus I for the indicated materials when $I \neq 0$

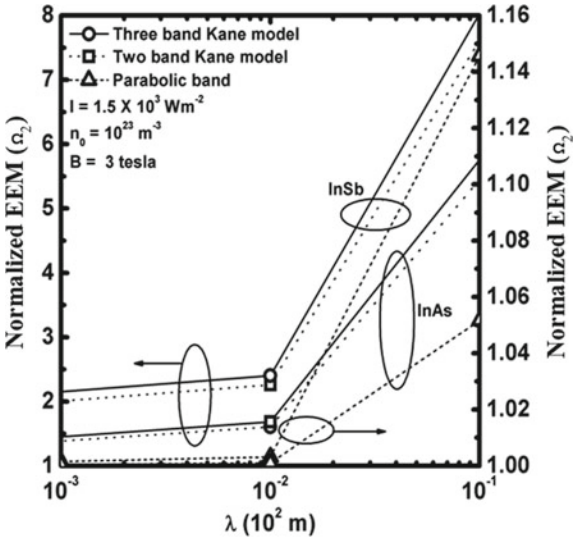

Fig. 1.36 Ω_2 versus λ for the indicated materials when $I \neq 0$

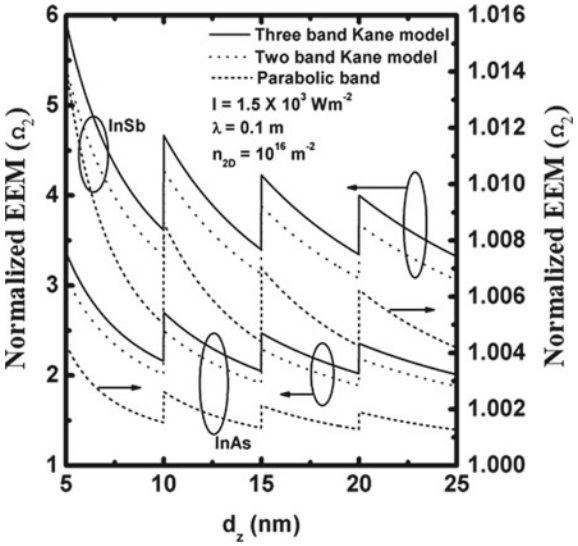

Fig. 1.37 Ω_2 versus d_z for QWs of the indicated materials when $I \neq 0$

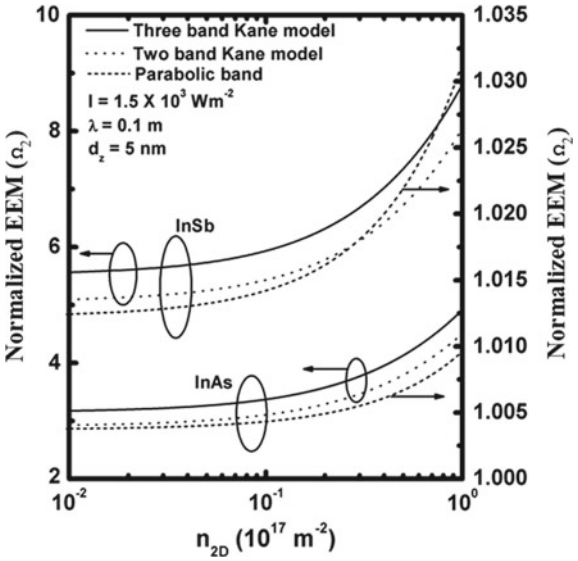

Fig. 1.38 Ω_2 versus n_{2D} for QWs of the indicated materials when $I \neq 0$

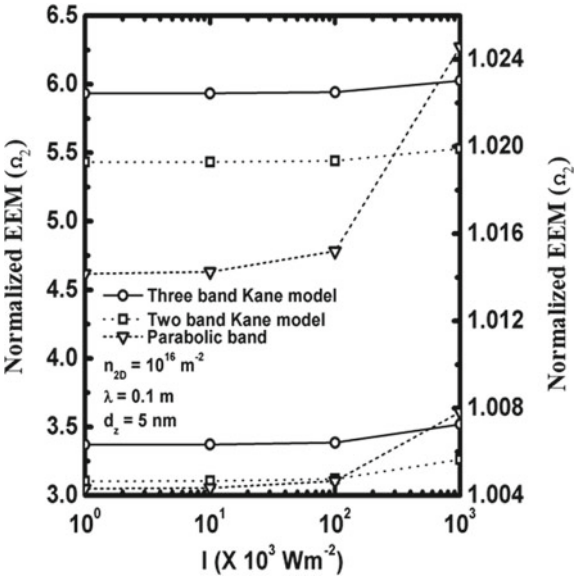

Fig. 1.39 Ω_2 versus I for QWs of the indicated materials when $I \neq 0$

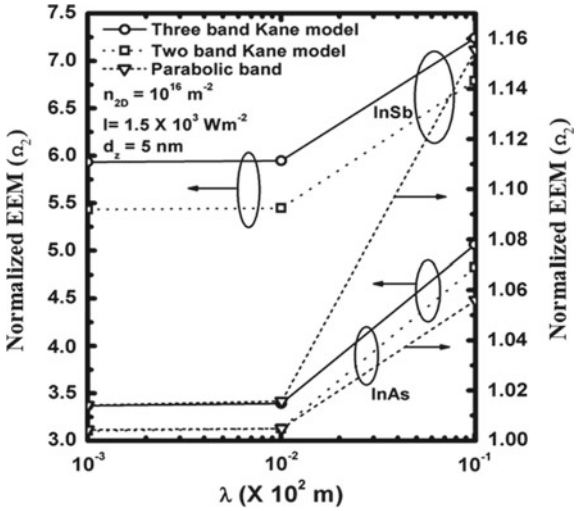

Fig. 1.40 Ω_2 versus λ for QWs of the indicated materials when $I \neq 0$

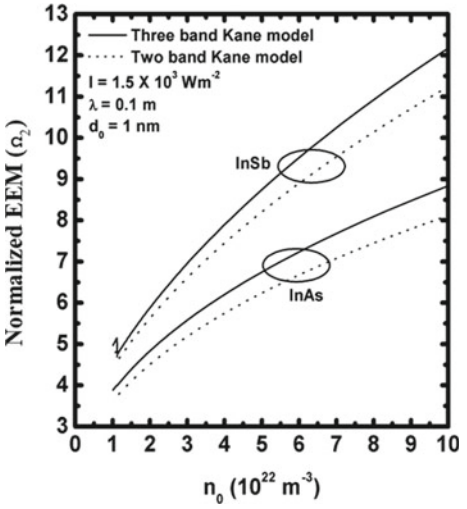

Fig. 1.41 Ω_2 versus n_0 for HD doping superlattices for $I \neq 0$ for the indicated materials and band models

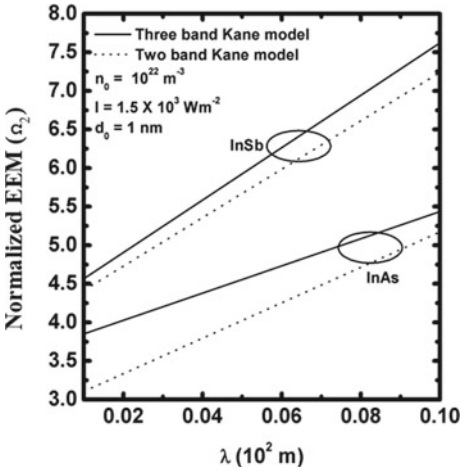

Fig. 1.42 Ω_2 versus λ for HD doping superlattices for the indicated materials and band models

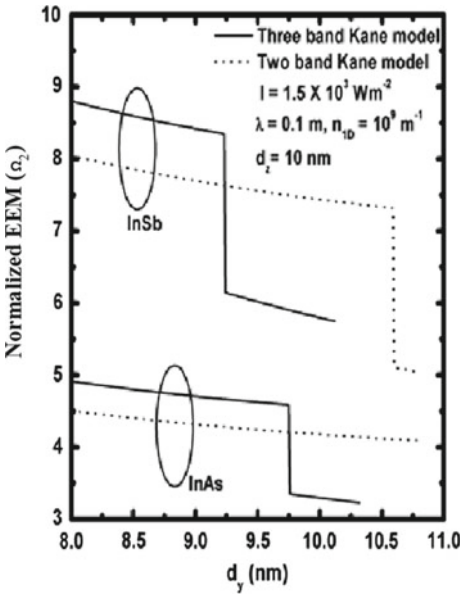

Fig. 1.43 Ω_2 versus d_y for HDNWs for the indicated materials and band models when $I \neq 0$

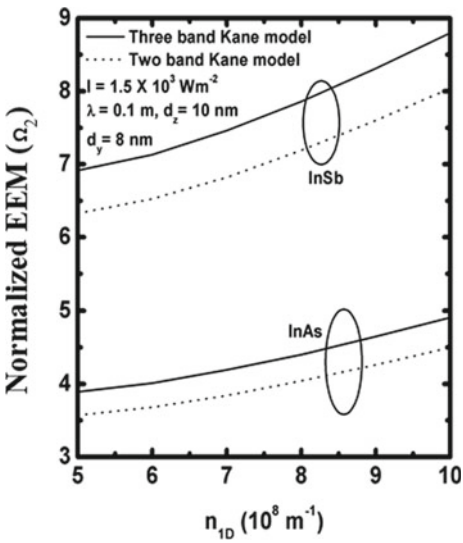

Fig. 1.44 Ω_2 versus n_{1D} for HDNWs for the indicated materials and band models when $I \neq 0$

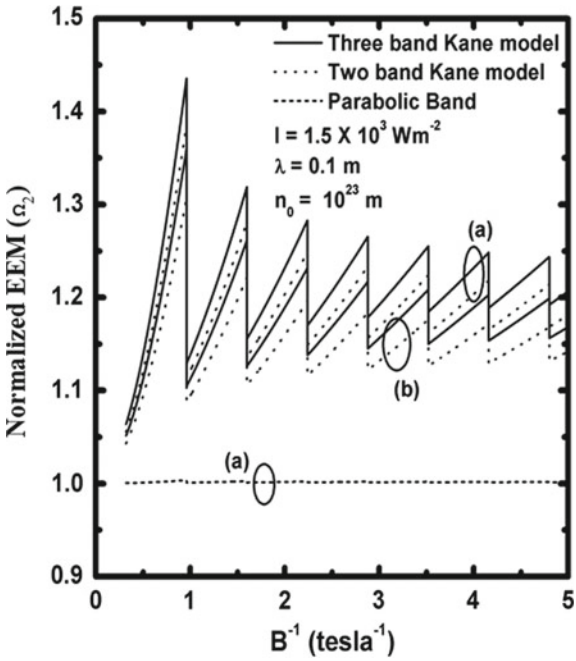

Fig. 1.45 Ω_2 for $n = 0$ and $I \neq 0$ versus $(1/B)$for GaAs/AlGaAs HDEMSL for the indicated band models

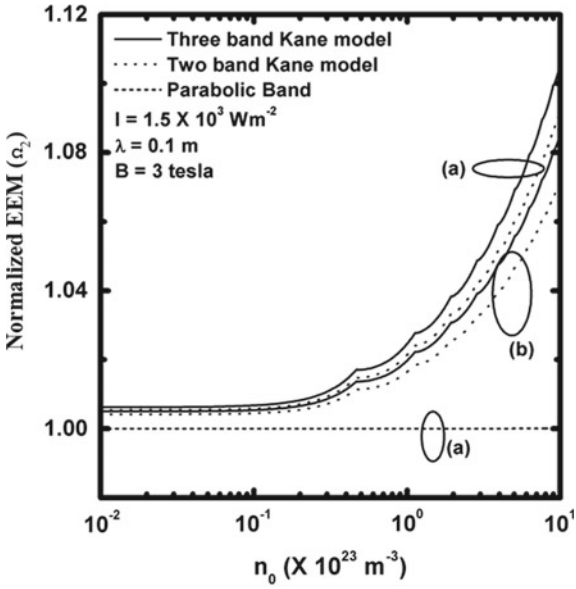

Fig. 1.46 Ω_2 for $n = 0$ and $I \neq 0$ versus $n_0$for GaAs/AlGaAs HDEMSL for the indicated band models

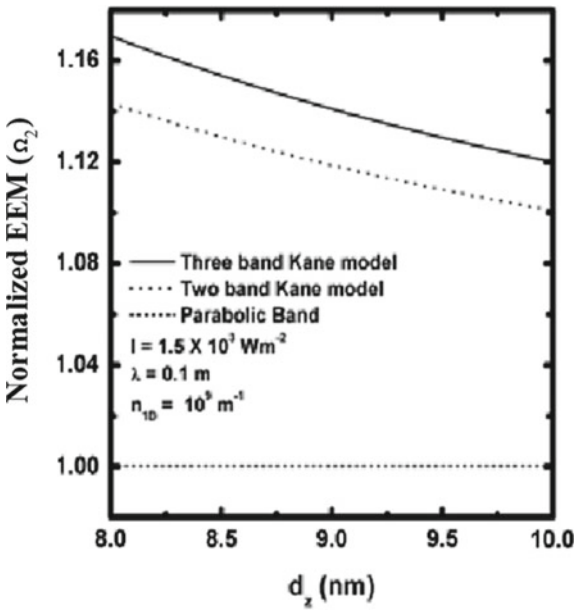

Fig. 1.47 Ω_2 for the quantum limits and $I \neq 0$ versus d_z for GaAs/AlGaAs HDNWEMSL for the indicated band models

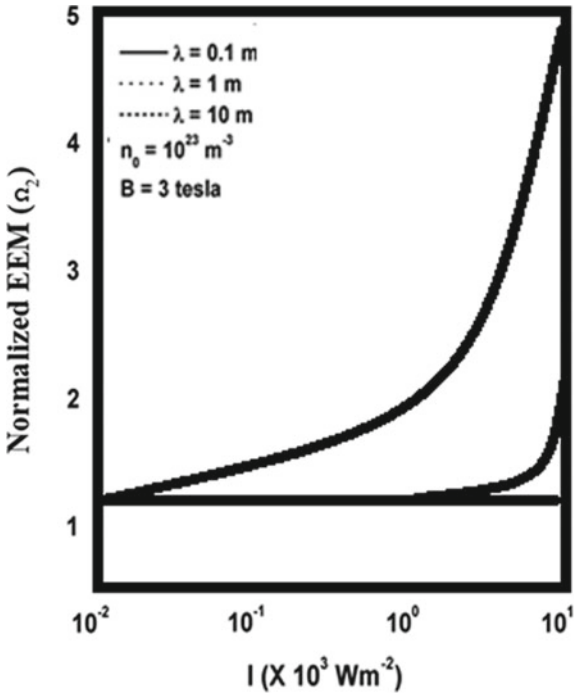

Fig. 1.48 Ω_2 for the quantum limit and $I \neq 0$ versus I for GaAs/AlGaAs HDSL with graded interfaces for two-band Kane model

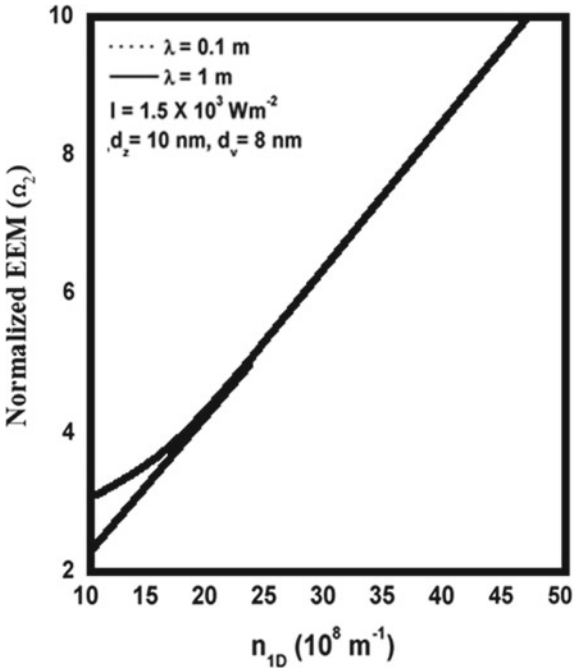

Fig. 1.49 Ω_2 for the quantum limits and $I \neq 0$ versus I for GaAs/AlGaAs HDNWSL with graded interfaces for two-band Kane model

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Chapter 2
The HUP and the ES in HD Kane-Type III-V and Opto-Electronic Materials under Intense Electric Field

2.1 Introduction

Under intense electric field, the different physical properties of modern nano-devices have been studied [1], using the idea that the $E - k$ relations are not altered under radiation fields which is fundamentally questionable. The electronic properties of electronic materials under strong electric field which change their DRs have mainly been investigated by Ghatak et al. [2–14].

We investigate chronologically the ES by applying HUP in accordance with the content of this chapter taking various III-V, ternary, quaternary systems and their low dimensional counterparts. Section 2.4 contains the result and discussions.

2.2 Mathematical Basis

2.2.1 The Bulk Materials

The DR under intense electric filed F_s becomes [6]

$$\overline{G}_4 k^2 = \left[e_1 E^4 + e_2 E^3 + e_3 E^2 + e_4 E + e_5 - \frac{e_6}{1 + CE} + e_7 (1 + CE)^{-2} \right] \quad (2.1)$$

where the notations are defined in [6].

Using Eqs. (2.1) and (1.7), the complex DR becomes

$$\overline{G}_4 k^2 = J_1(\xi_{20}) + i J_2(\xi_{20}) \quad (2.2)$$

where, $\xi_{20} = (E, c, \eta_g, F)$.

Using Eq. (2.2), the EEM becomes

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$$m^*(\xi_{20})/m_c = J_1'(\xi_{20}) \tag{2.3}$$

The ES can be written from Eq. (2.2) using HUP as

$$n_0 = \text{R. P. } \overline{n}_0 \tag{2.4}$$

where, $\overline{n}_0 = \overline{G}_2[[J_1(\xi_{21}) + iJ_2(\xi_{21})]^{3/2}]$, $\xi_{21} = (E_{\text{Fe1}}, c, \eta_g, F)$ and E_{Fe1} is the corresponding Fermi energy.

2.2.2 The Magnetic Quantization

The HDDR becomes

$$k_x^2 = w_{11}(\xi_{22}) \tag{2.5}$$

where, $\xi_{22} = (E, F, n, \eta_g)$ and $w_{11}(\xi_{22}) = \frac{2m_c}{\hbar^2}\left[J_1(\xi_{20}) + iJ_2(\xi_{20}) - \left(n + \frac{1}{2}\right)\hbar\omega_0\right]$.

The EEM and n_0 become

$$m^*(\xi_{20})/m_c = J_1'(\xi_{20}) \tag{2.6}$$

$$n_0 = R.P.\ \overline{n}_0 \tag{2.7}$$

where, $\overline{n}_0 = \overline{G}_3 \sum_{n=0}^{n_{\max}} \left[\left[[J_1(\xi_{23}) + iJ_2(\xi_{23})] - \left(n + \frac{1}{2}\right)\hbar w_0\right]^{1/2}\right]$ and $\xi_{23} = (E_{\text{Fe2}}, c, \eta_g, F)$.

2.2.3 The Quantum Wells

The DR assumes the form

$$\overline{G}_5 + \overline{G}_4 k_s^2 = [J_1(\xi_{20}) + iJ_2(\xi_{20})] \tag{2.8}$$

The EEM and n_0 can, respectively, be written as

$$m^*(\xi_{20})/m_c = J_1'(\xi_{20}) \tag{2.9}$$

$$n_0 = R.\ P.\ \overline{n}_0 \tag{2.10}$$

where, $\overline{n}_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} \left[\left[[J_1(\xi_{24}) + i\,J_2(\xi_{24})] - \overline{G}_5\right]\right]$ and $\xi_{24} = E_{\mathrm{Fe3}}, c, \eta_{\mathrm{g}}, F.$

2.2.4 The Nanowires

The DR assumes the form

$$\overline{G}_{11} + \overline{G}_4 k_x^2 = [J_1(\xi_{20}) + i\,J_2(\xi_{20})] \tag{2.11}$$

The EEM and n_0 become

$$m^*(\xi_{20})/m_c = J_1'(\xi_{20}) \tag{2.12}$$

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.13}$$

where, $\overline{n}_0 = \overline{G}_{12} \sum_{n_y=1}^{n_{y\max}} \sum_{n_z=1}^{n_{z_z\max}} \left[\left[[J_1(\xi_{25}) + i\,J_2(\xi_{25})] - G_2\left(n_{\overline{y}}, n_{\overline{z}}\right)\right]^{1/2}\right]$ and $\xi_{25} = E_{\mathrm{Fe4}}, c, \eta_g, F.$

2.2.5 The Quantum Dots

The DR assumes the form

$$\overline{G}_7 = [J_1(\xi_{26}) + i\,J_1(\xi_{26})] \tag{2.14}$$

where, $E_{n_z 18,3\mathrm{HD}}$ the sub-band energy and $\xi_{26} = (E_{n_z 18,3\mathrm{HD}}, c, \eta_g, F)$ the n_0 becomes

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.15}$$

where, $\overline{n}_0 = \overline{G}_8 F_{-1}(\eta_{1e})$
in which $\eta_{1e} = (k_B T)^{-1}(E_{\mathrm{Fe5}} - E_{n_z 18,3\mathrm{HD}})$.

2.2.6 The Magneto-Size Quantization

The DR assumes the form

$$\overline{G}_9 = [J_1(\xi_{27}) + i\,J_2(\xi_{27})] \tag{2.16}$$

where, $\xi_{27} = (E_{n_z 18,4\mathrm{HD}}, c, \eta_g, F)$.

Using Eq. (2.16), the n_0 becomes

$$n_0 = R.P.\ \overline{n}_0 \tag{2.17}$$

where, $\overline{n}_0 = \overline{G}_{10} F_{-1}(\eta_{2e})$

in which $\eta_{2e} = (k_B T)^{-1}(E_{\mathrm{Fe6}} - E_{n_z 18,4\mathrm{HD}})$.

2.2.7 The Doping Superlattices

The DR in HDDSL can be written as

$$\omega_{91HD1}(\xi_{20}) = \left(\frac{n_s |e|^2}{d_0 \varepsilon_{sc}[J_1(\xi_{20}) + i J_2(\xi_{20})]' m_c}\right)^{\frac{1}{2}} \tag{2.18}$$

where, $\eta_{2e} = (k_B T)^{-1}(E_{\mathrm{Fe6}} - E_{n_z 18,4\mathrm{HD}})$.

The EEM and n_0 become

$$m^*(\xi_{20}, n_i)/m_c = R.P.[P_{3\mathrm{HDL3}}(\xi_{20}, n_i)]' \tag{2.19}$$

$$n_0 = R.\ P.\ \overline{n}_0 \tag{2.20}$$

where, $[P_{3\mathrm{HDL3}}(\xi_{20}, n_i)] = \left[[J_1(\xi_{20}) + i J_2(\xi_{20})] - \left(n_i + \frac{1}{2}\right)\hbar\omega_{91\mathrm{HD1}}(\xi_{20})\right]$,

$$\overline{n}_0 = \overline{G}_4 \left[\sum_{n_i=0}^{n_{i\max}} \left[J_1(\xi_{28}) + i J_2(\xi_{28}) - \left(n_i + \frac{1}{2}\right)\hbar\omega_{91\mathrm{HD1}}(\xi_{28})\right]\right] \text{ and}$$

$\xi_{28} = \left(E_{\mathrm{Fel1}}, c, \eta_g, F\right)$.

2.2.8 The Magneto Doping Superlattices

The magneto DR and n_0 in HDDSL become

$$[J_1(\xi_{29}) + i J_2(\xi_{29})] = \left(n_i + \frac{1}{2}\right)\hbar\omega_{91\mathrm{HD1}}(\xi_{29}) + \left(n + \frac{1}{2}\right)\hbar\omega_0 \tag{2.21}$$

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.22}$$

where, $\xi_{29} = (E_{555}, c, \eta_g, F)$,

$$\overline{n}_0 = \overline{G}_{15} F_{-1}(\overline{\eta}_{3e})$$

in which $\overline{\eta}_{3e} = (k_B T)^{-1}(E_{\text{Fe11}} - E_{555})$.

2.2.9 The Quantum Well Effective Mass Superlattices

The DR becomes

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{18\text{HD1}}(E, k_y, k_{z,} F))\}^2 - k_\perp^2\right] \tag{2.23}$$

where the notation is defined in [6].

The DR in QWHDEMSLs assumes the form

$$\left(\frac{n_x \pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{18\text{HD1}}(E, k_y, k_{z,} F))\}^2 - k_\perp^2\right] \tag{2.24}$$

Using Eq. (2.24), the EEM and E_{600} become

$$m^*(k_\perp, E, F) = \frac{\hbar^2}{L_0^2}\left[\frac{\cos^{-1}[f_{18\text{HD1}}(E, k_y, k_z, F)] f'_{18\text{HD1}}(E, k_y, k_z, F)}{\sqrt{1 - f_{18\text{HD1}}^2(E, k_y, k_z, F)}}\right] \tag{2.25}$$

$$\left(\frac{n_x \pi}{d_x}\right) = \left[\frac{1}{L}\{\cos^{-1}(f_{18\text{HD1}}(E_{600}, k_y, k_{z,} F))\}\right] \tag{2.26}$$

The ES has to be calculated numerically.

2.2.10 The Magneto Effective Mass Superlattices

Using Eq. (2.27), the magneto DR in HDEMSLs becomes

$$k_x^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{18\text{HD2}}(E, n, F))\}^2 - \frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right] \tag{2.27}$$

where the notations are defined in elsewhere [6].

THE EEM and n_0 become

$$m^*(n, E, F) = \frac{\hbar^2}{L_0^2}\left[\frac{\cos^{-1}[f_{18HD2}(E, n, F)]f'_{18HD2}(E, n, F)}{\sqrt{1 - f^2_{18HD2}(E, n, F)}}\right] \tag{2.28}$$

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.29}$$

where, $\overline{n}_0 = \overline{G}_{34}[[\omega_{100}(E_{Fe12}, n, F)]]$, in which E_{Fe12} is the corresponding Fermi energy and

$$\omega_{100}(E_{Fe12}, n, F) = \left[\left[\frac{1}{L_0^2}\{\cos^{-1}(f_{18HD2}(E_{Fe12}, n, F))\}^2 - \frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right]\right]^{\frac{1}{2}}.$$

2.2.11 The Nanowire Effective Mass Superlattices

Using (2.27), the DR can be expressed as

$$k_x^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{18HD1}\left(E, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_y}, F\right)\right)\right\}^2 - \overline{G}_{36}\right] \tag{2.30}$$

where the notation is defined in [6].

The EEM becomes

$$m^*(\overline{Z}_1, F) = \frac{\hbar^2}{L_0^2}\left[\frac{\cos^{-1}[f_{18HD1}(\overline{Z}_1, F)]f'_{18HD1}(\overline{Z}_1, F)}{\sqrt{1 - f^2_{18HD1}(\overline{Z}_1, F)}}\right] \tag{2.31}$$

where, $\overline{Z}_1 = \left(E, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_z}\right)$.

The ES becomes

$$n_0 = \frac{2g_v}{\pi}\left[\sum_{n_y=1}^{n_{ymax}}\sum_{n_z=1}^{n_{zmax}}\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{18HD1}\left(E_{Fe40}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_y}, F\right)\right)\right\}^2 - \overline{G}_{36}\right]^{1/2}\right]. \tag{2.32}$$

2.2.12 The Quantum Dot Effective Mass Superlattices

Using (2.27), the DR in QDHDEMSLs can be expressed as

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_{18\mathrm{HD1}}\left(\overline{E}_{600}, \frac{n_y\pi}{d_y}, \frac{n_z\pi}{d_y}, F\right)\right)\right\}^2 - \overline{G}_{36}\right] \tag{2.33}$$

where $\overline{E}_{600}$ is the sub-band energy.

Using (2.33), the n_0 can be written as

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.34}$$

where, $\overline{n}_0 = \overline{G}_{30}[F_{-1}(\eta_{6e})]$

in which $\eta_{6e} = (k_B T)^{-1}(E_{\mathrm{Fe13}} - \overline{E}_{600})$ and E_{Fe13} is the corresponding Fermi energy.

2.2.13 The Magneto Quantum Well Effective Mass Superlattices

The DR in HDEMSLs becomes

$$\left(\frac{n_x\pi}{d_x}\right)^2 = \left[\frac{1}{L_0^2}\{\cos^{-1}(f_{18\mathrm{HD2}}(E_{601}, n, F))\}^2 - \frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right] \tag{2.35}$$

where E_{601} is the sub-band energy.

Using Eq. (2.35), the n_0 becomes

$$n_0 = \overline{G}_{35}[F_{-1}(\eta_{7e})] \tag{2.36}$$

where, $\eta_{7e} = (k_B T)^{-1}(E_{\mathrm{Fe14}} - E_{601})$.

2.2.14 The Quantum Well Superlattices with Graded Interfaces

The DR becomes [14]

$$k_z^2 = G_{8,19} + i\,H_{8,19} \tag{2.37}$$

where the notations are defined elsewhere [14].

The simplified DR in the corresponding QWs case becomes

$$\left(\frac{n_z\pi}{d_z}\right)^2 = G_{8,19} + i\,H_{8,19} \tag{2.38}$$

The EEM and n_0 have to be Calculated numerically.

2.2.15 The Nanowire Superlattices with Graded Interfaces

The DR becomes [14]

$$k_z^2 = G_{8,20} + i\,H_{8,20} \tag{2.39}$$

where the notation is defined in elsewhere [14].

The EEM and n_0 become

$$m^*(E, n_y, n_z\eta_g, F) = \frac{\hbar^2}{2}(G'_{8,20}) \tag{2.40}$$

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.41}$$

where, $\overline{n}_0 = \frac{2g_v}{\pi}\left[\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\left[\sqrt{G_{8,20}+i\,H_{8,20}}\right]\Big|_{E=E_{\text{Fel5}}}\right]$.

2.2.16 The Quantum dot Superlattices with Graded Interfaces

The DR is given by

$$\left(\frac{n_z\pi}{d_z}\right)^2 = [G_{8,20} + i\,H_{8,20}]_{E=E_{620}} \tag{2.42}$$

where E_{620} is the totally quantized sub-band energy.

The n_0 becomes

$$n_0 = R.\,P.\,\overline{n}_0 \tag{2.43}$$

where, $\overline{n}_0 = \overline{G}_{30} F_{-1}(\overline{\eta}_{8e})$

in which $\overline{\eta}_{8e} = (k_B T)^{-1}(E_{\text{Fe18}} - E_{620})$ and E_{Fe18} is the Corresponding Femi energy.

2.2.17 The Magneto Superlattices with Graded Interfaces

The magneto DR becomes [14]

$$k_z^2 = G_{8,19n} + i H_{8,19n} \tag{2.44}$$

The EEM and n_0 become

$$m^*(E, n, \eta_g) = \frac{\hbar^2}{2}(G'_{8,19n}) \tag{2.45}$$

$$n_0 = R.P.\ \overline{n}_0 \tag{2.46}$$

where, $\overline{n}_0 = \frac{g_v eB}{\pi^2 \hbar}\left[\sum_{n=0}^{n_{\max}} \sqrt{[G_{8,19n} + i H_{8,19n}]}\right]\Big|_{E=E_{\text{Fe19}}}$

in which E_{Fe92} is the Corresponding Femi energy.

2.2.18 The Magneto Quantum Well Superlattices with Graded Interfaces

The magneto DR is given by

$$\left(\frac{n_z \pi}{d_z}\right)^2 = [G_{8,19n} + i H_{8,19n}]_{E=E_{650}} \tag{2.47}$$

The ES becomes

$$n_0 = R.P.\ \overline{n}_0 \tag{2.48}$$

where, $\overline{n}_0 = \overline{G}_{10}[F_{-1}(\overline{\eta}_{16e})]$

in which $\overline{\eta}_{16e} = (k_B T)^{-1}(E_{\text{Fe20}} - E_{650})$.

2.3 Result and Discussions

We have plotted the normalized Fermi energy (Ω_1) in this case with different physical variables in Figs. 2.1–2.14.

From the figures we note the following points:

1. From Figs. 2.1 and 2.2, it appears that Ω_1 exhibits monotonic increasing dependence with respect to E_0 and n_0 for all the materials in two different ways, and from Fig. 2.3, we observe that Ω_1 increases with decreasing x.
2. From Fig. 2.4, we note that Ω_1 decreases with increasing d_y in oscillatory manner in QWs of the considered materials.

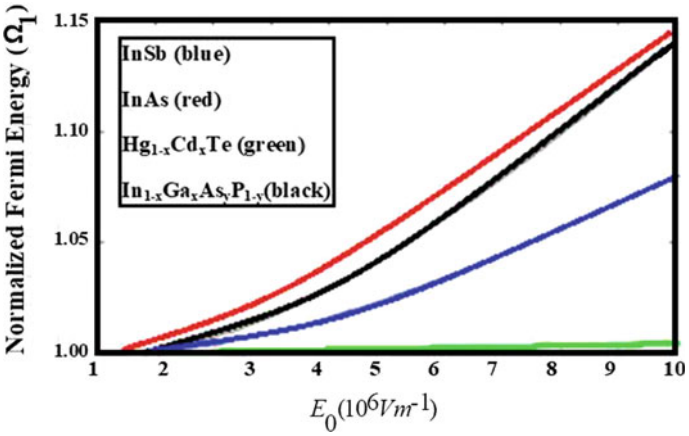


Fig. 2.1 Ω_1 against E_0 for the indicated compound

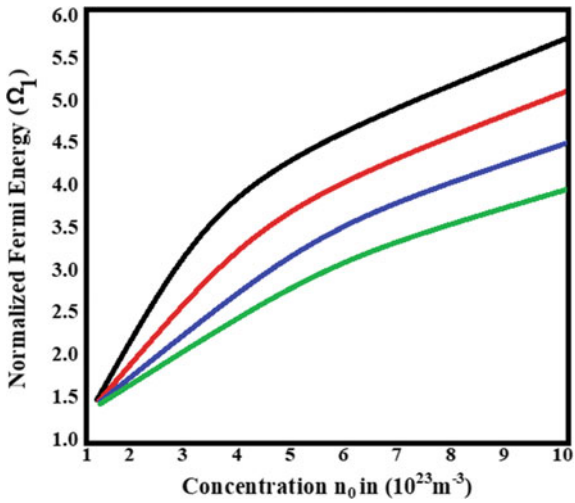


Fig. 2.2 The Ω_1 versus n_0 for all cases of Fig. 2.1

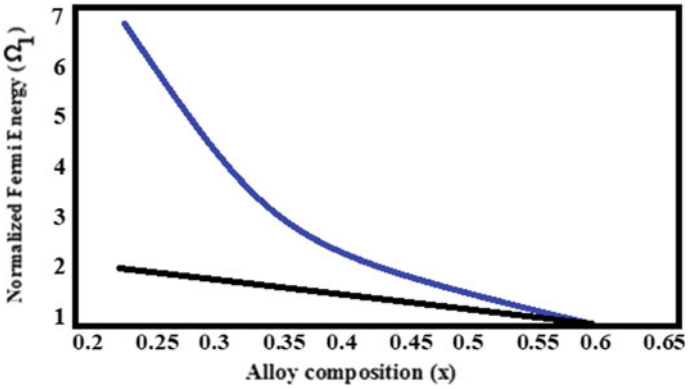
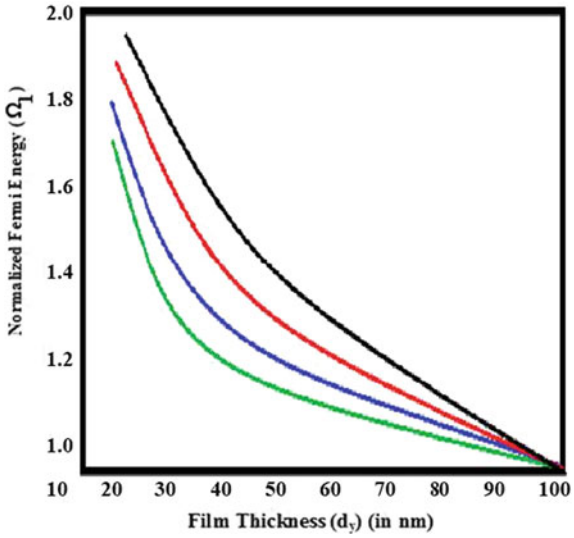


Fig. 2.3 The Ω_1 versus x for the indicated materials

Fig. 2.4 The Ω_1 versus d_y in QWs for all the materials of Fig. 2.1



3. From Fig. 2.5, we note that for QWs, Ω_1 increases with increasing n_0.
4. From Fig. 2.6, we observe that for QWs, Ω_1 increases with increasing E_0 in quantized steps which exhibits the signature of quantum confinement.
5. From Fig. 2.7, we can write that Ω_1 increases with decreasing x in different manners for QWs of the indicated materials.
6. Fig. 2.8 exhibits the fact that Ω_1 changes with changing d_y in sharply defined oscillatory manners in NWs of the considered materials.
7. From Fig. 2.9, we note that for NWs, Ω_1 enhances with enhanced n_0 in an oscillatory way.
8. From Fig. 2.10, we observe that for NWs, the Ω_1 increases with increasing E_0

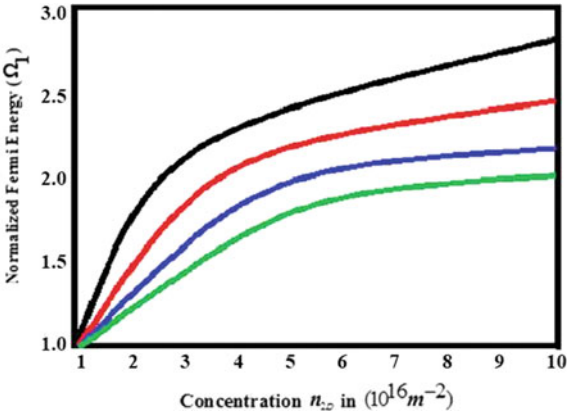


Fig. 2.5 The Ω_1 versus n_0 in QWs for all the materials of Fig. 2.1

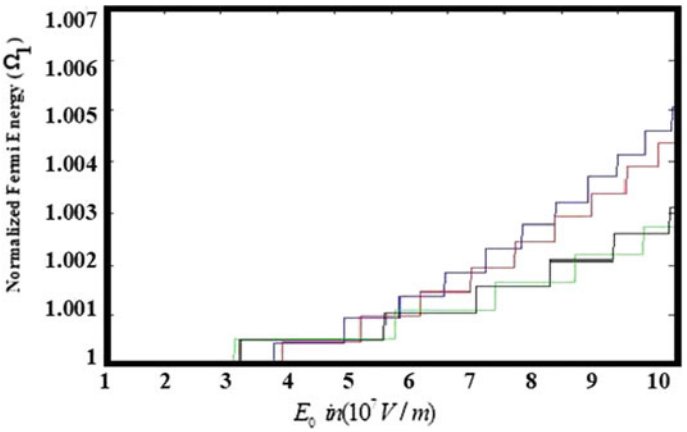


Fig. 2.6 The Ω_1 versus E_0 in QWs for all the materials of Fig. 2.1

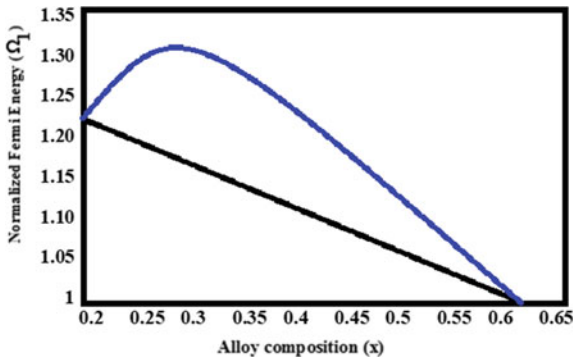


Fig. 2.7 The Ω_1 versus x in QWs of the indicated materials

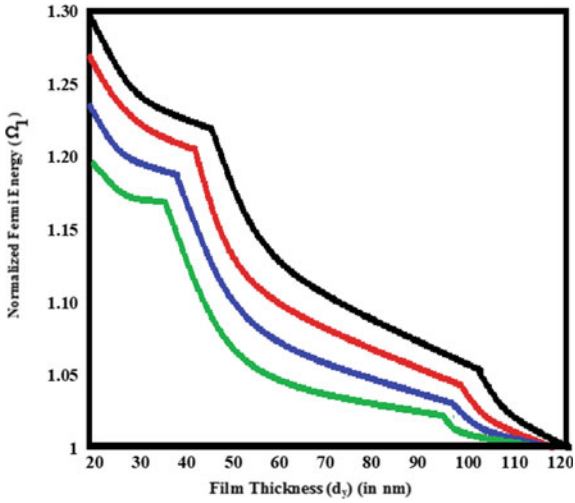


Fig. 2.8 The Ω_1 versus d_y in NWs for all the materials of Fig. 2.1

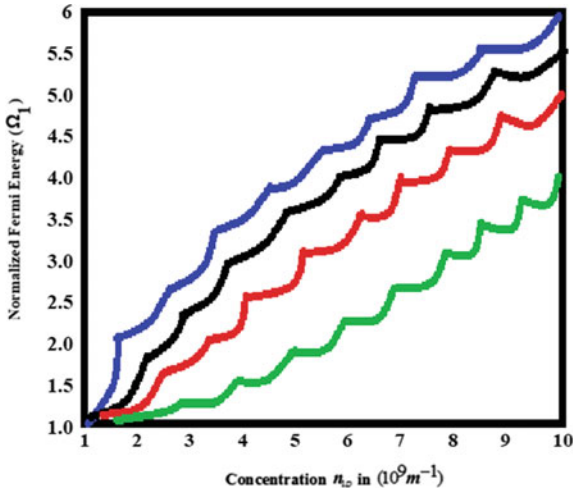


Fig. 2.9 The Ω_1 against n_0 in NWs for the materials of Fig. 2.1

9. From Fig. 2.11, we can write that Ω_1 increases with decreasing x in different manners for NWs of the indicated materials.
10. From Fig. 2.12, we observe that Ω_1 oscillates with $(1/B)$ due to SdH effect.
11. From Fig. 2.13, we note that magneto Ω_1 increases with increasing electric field.
12. From Fig. 2.14, we can write that magneto Ω_1 increases with decreasing x in different manners for the indicated materials.

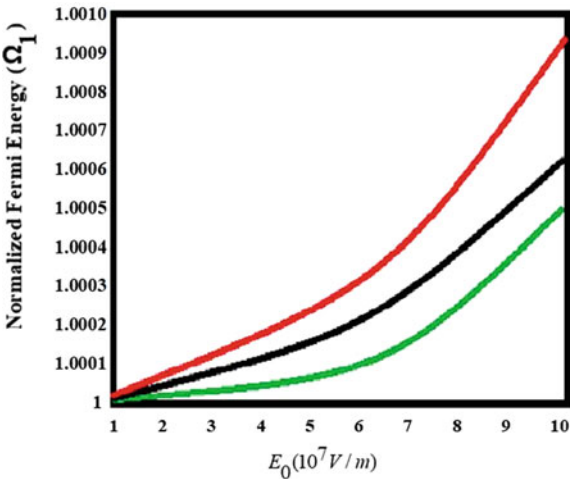


Fig. 2.10 The Ω_1 versus E_0 in NWs for all the materials of Fig. 2.1

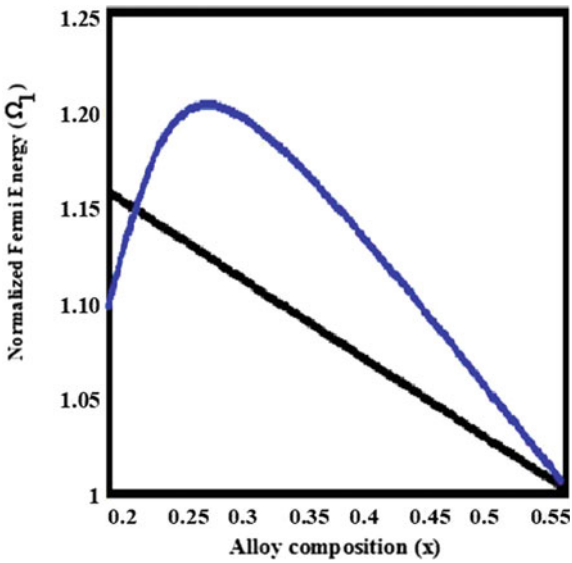


Fig. 2.11 The Ω_1 versus x in NWs for the indicated materials

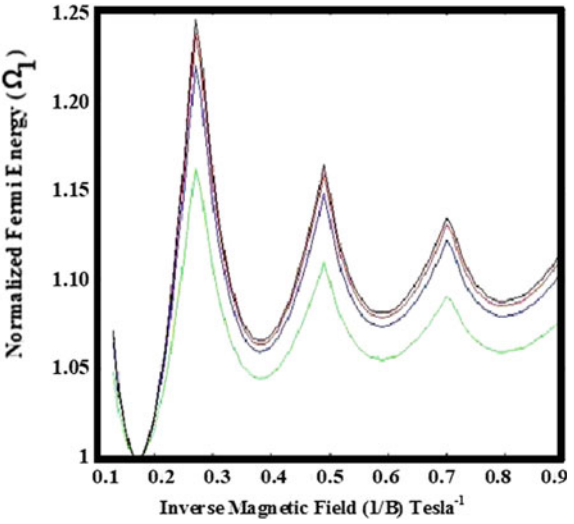


Fig. 2.12 Ω_1 against B^{-1} for all the materials of Fig. 2.1

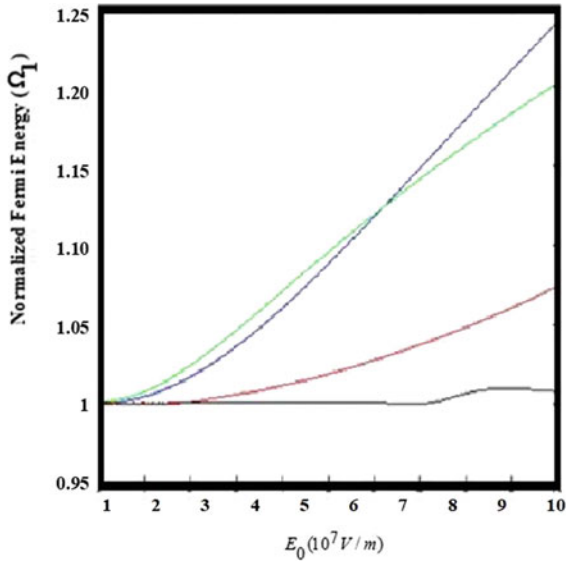


Fig. 2.13 The magneto Ω_1 versus E_0 for all the materials of Fig. 2.1

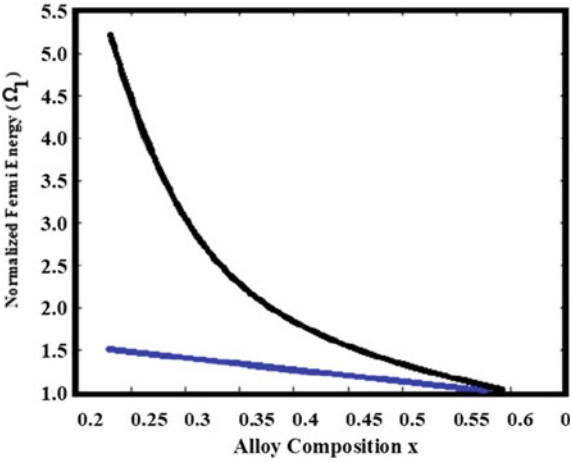


Fig. 2.14 The magneto Ω_1 versus x for all the indicated materials

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Chapter 3
The HUP and the ES in Quantum Wells (QWs) of HD Non-parabolic Materials

3.1 Introduction

The 2D electron transport in quantum wells (QWs) has been extensively studied [1–77]. Here, we study the ES in QWHD Non-Parabolic compounds by applying the HUP directly. We study chronologically the ES in accordance with the content of this chapter taking QWs of HD nonlinear optical, III-V, II-VI, IV-VI, stressed Kane-type materials, Te, Gallium Phosphide, Platinum Antimonide, Bismuth Telluride, Gallium Antimonide and II-V, respectively, which find extensive uses in various device applications [74, 78–159]. Section 3.3 contains the result and discussions.

3.2 Mathematical Basis

3.2.1 The Nonlinear Optical Materials

Here, the 2*D* DR becomes [122]

$$\frac{\hbar^2(n_z\pi/d_z)^2}{2m_{\|}^{*}T_{21}(E,\eta_g)}+\frac{\hbar^2k_s^2}{2m_{\perp}^{*}T_{22}(E,\eta_g)}=1 \tag{3.1}$$

Using Eq. (3.1), the EEM becomes [122]

$$m^{*}(E_{F1\mathrm{HD}},\eta_g,n_z)=m_{\perp}^{*}[R.P.\,T_{1D}'(E_{F1\mathrm{HD}},\eta_g,n_z)] \tag{3.2}$$

where, $E_{F1\mathrm{HD}}$ is the corresponding Fermi energy and $T_{ID}(E_{F1\mathrm{HD}},\ \eta_g,\ n_z)=\left[1-\frac{\hbar^2(n_z\pi/d_z)}{2m_{\|}^{*}T_{21}(E_{F1\mathrm{HD}},\eta_g)}\right]T_{22}(E_{F1\mathrm{HD}},\eta_g)$

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Although the importance of the EEM is already noted [123–144], we observe that the EEM is the function of η_g due to which the EEM exists in the band gap.

By using HUP n_0 becomes

$$n_0 = R.P.\overline{n}_0 \tag{3.3}$$

where,

$$\overline{n}_0 = z_1 \sum_{n_z=1}^{n_{\max}} [[T_{1D}(E_{F1\mathrm{HD}}, \eta_{g,} n_z)]] \tag{3.4}$$

and $z_1 = \frac{\overline{G}_6 m_\perp^*}{m_c}$

When $\eta_g \to 0$, the $2D$ EEM and the ES become

$$m^*(E_{Fs}, n_z) = \left(\frac{\hbar^2}{2}\right) T'_{51}(E_{Fs}, n_z) \tag{3.5}$$

and

$$n_0 = z_2 \sum_{n_z=1}^{n_{z\max}} T_{51}(E_{FS}, n_z) \tag{3.6}$$

where, $z_2 = \left(\frac{g_v}{2\pi}\right)$

3.2.2 The III-V Materials

Here, the $2D$ DR becomes

$$\overline{G}_5 + \overline{G}_4 k_s^2 = T_{31}(E, \eta_g) + i T_{32}(E, \eta_g) \tag{3.7}$$

The EEM and ES become

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z) = m_c[T'_{31}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.8}$$

$$n_0 = R.P.\overline{n}_0 \tag{3.9}$$

where,

$$\overline{n}_0 = \overline{G}_6 \sum_{n_z-1}^{n_{\max}} [[T_{5D}(E_{F1\mathrm{HD}}, \eta_g, n_z)]]. \tag{3.10}$$

In which $E_{F1\mathrm{HD}}$ is the corresponding Fermi energy

and $T_{5D}(E_{F1\mathrm{HD}}, \eta_g, n_z) = [T_{31}(E_{F1\mathrm{HD}}, \eta_g) + iT_{32}(E_{F1\mathrm{HD}}, \eta_g) - \overline{G}_5]$

For $\eta_g \to 0$, the $2D$ DR, EEM and n_0 become

$$\overline{G}_4 k_s^2 + \overline{G}_5 = I_{11}(E) \tag{3.11}$$

$$m^*(E_{Fs})/m_c = \{I_{11}(E_{Fs})\}' \tag{3.12}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{53}(E_{Fs}, n_z)] \tag{3.13}$$

where $T_{53}(E_{Fs}, n_z) \equiv [I_{11}(E_{Fs}) - \overline{G}_5]$

Under the inequalities $\Delta >> E_{g_0}$ or $\Delta << E_{g_0}$, (3.8), (3.9) and (3.10) assume the forms

$$\overline{G}_5 + \overline{G}_4 k_s^2 = \gamma_2(E, \eta_g) \tag{3.14}$$

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z)/m_c = [\gamma_2'(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.15}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{7D}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.16}$$

where $T_{7D}(E_{F1\mathrm{HD}}, \eta_g, n_z) = [\gamma_2(E_{F1\mathrm{HD}}, \eta_g) - \overline{G}_5]$

The DR when $\eta_g \to 0$ in QWs of HDS with unperturbed wide band gap dispersion law is given by

$$\overline{G}_5 + \overline{G}_4 k_s^2 = \gamma_3(E, \eta_g) \tag{3.17}$$

The EEM and n_0 become

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z)/m_c = [\gamma_3'(E_{F1\mathrm{HD}}, \eta_g)] \tag{3.18}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{9D}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.19}$$

where $T_{9D}(E_{F1\mathrm{HD}}, \eta_g) = [\gamma_3(E_{F1\mathrm{HD}}, \eta_g) - \overline{G}_5]$.

3.2.3 The II-VI Materials

Here, the 2*D* DR becomes [121]

$$\gamma_3(E, \eta_g) = a_0' k_s^2 + b_0' \left(\frac{\pi n_z}{d_z}\right)^2 \pm \overline{\lambda}_0 k_s \tag{3.20}$$

From Eq. (3.18), the EEM becomes

$$m^*(E_{F1HD}, n_z, \eta_g) = m_\perp^* \left[1 \mp \frac{(\bar{\lambda}_0)}{\left[(\bar{\lambda}_0)^2 - 4a_0' b_0' \left(\frac{n_z \pi}{d_z}\right)^2 + 4a_0' \gamma_3(E_{F1HD}, \eta_g)\right]^{1/2}}\right] \gamma_3'(E_{F1HD}, \eta_g) \tag{3.21}$$

The $E_{n_z D14}$ becomes

$$\gamma_3(E_{n_z D14}, \eta_g) = b_0' \left(\frac{\pi n_z}{d_z}\right)^2 \tag{3.22}$$

The (n_0) is given by

$$n_0 = z_1 \sum_{n_Z=1}^{n_{z\max}} (\gamma_3(E_{F1\mathrm{HD}}, \eta_g) - E_{n_Z D_{14}} + (\overline{\lambda})^2 m_\perp^* \hbar^{-2}). \tag{3.23}$$

3.2.4 The IV-VI Materials

(a) **The Model of Dimmock** [147]

The n_0 and EEM become [122]

$$n_0 = z_2 \sum_{n_z=1}^{n_{\max}} [T_{55\mathrm{HD}}(E_{F1\mathrm{HD}}, n_z)] \tag{3.24}$$

and

$$m^*(E, n_z) = \frac{\hbar^2}{2}[\theta_{5\mathrm{HD}}(E, n_z)]\Big|_{E=E_{F1\mathrm{HD}}} \tag{3.25}$$

where $T_{55\mathrm{HD}}(E_{F1\mathrm{HD}}, n_z) \equiv \frac{A_{\mathrm{HD}}(E_{F1\mathrm{HD}}, n_z)}{\pi}$
under the condition $\eta_g \to 0$ is given by [122]

$$n_0 = \left(\frac{g_v}{2\pi^2}\right)[M_{A4}(E_{F_b})] \tag{3.26}$$

(b) **The Model of Bangert and Kastner** [148]

Here, the 2*D* DR becomes

$$k_s^2 = F_6(E, \eta_g, n_z) \tag{3.27}$$

where, $F_6(E, \eta_g, n_z) = \left[\frac{[1-F_2(E,\eta_g)(n_z\pi/d_z)^2]}{F_1(E,\eta_g)}\right]$

The EEM becomes

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z) = \frac{\hbar^2}{2} R.P.[F_6'(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.28}$$

The n_0 becomes

$$n_0 = R.P.\overline{n}_0 \tag{3.29}$$

where,

$$\overline{n}_0 = z_2\left[\sum_{n_z=1}^{n_{z\max}} [F_6(E_{F1\mathrm{HD}}, \eta_g, \mathrm{n}_z)]\right] \tag{3.30}$$

When $\eta_g \to 0$, the 2*D* DR becomes [122]

$$k_s^2 = F_{16}(E, n_z) \tag{3.31}$$

where $F_{16}(E, n_z) = \left[\frac{[1-F_{12}(E)(n_z\pi/d_z)^2]}{F_{11}(E)}\right]$

From Eq. (3.31), the EEM becomes [6]

$$m^*(E_{F_s}, n_z) = \frac{\hbar^2}{2}[F_{16}'(E_{F_s}, n_z)] \tag{3.32}$$

The $\overline{n}_0$ can be expressed as

$$\overline{n}_0 = z_1 \left[\sum_{n_z=1}^{n_{z\max}} [F_{16}(E_{F_s}, \mathrm{n}_z)] \right] \tag{3.33}$$

3.2.5 The Stressed Kane-Type Materials

Here, the 2*D* DR in HDQWs becomes [122]

$$P_{11}(E, \eta_g)k_x^2 + Q_{11}(E, \eta_g)k_y^2 + S_{11}(E, \eta_g)\left(\frac{\pi n_z}{d_z}\right)^2 = 1 \tag{3.34}$$

The EEM and n_0 become [122]

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z) = \frac{\hbar^2}{2} A'_{56}(E_{F1\mathrm{HD}}, \eta_g, n_z) \tag{3.35}$$

$$n_0 = z_2 \sum_{n_z=1}^{n_{\max}} [T_{57\mathrm{HD}}(E_{F1\mathrm{HD}}, \eta_g, \eta_z)] \tag{3.36}$$

where $A_{56}(E, \eta_g, n_z) = \frac{\pi[1-S_{11}(E,\eta_g)(\eta_z\pi/d_z)^2]}{\sqrt{P_{11}(E,\eta_g)Q_{11}(E,\eta_g)}}$ and $T_{57\mathrm{HD}}(E_{F1\mathrm{HD}}, \eta_g, \eta_z) \equiv A_{56\mathrm{HD}}(E_{F1\mathrm{HD}}, \eta_g, \eta_z)$

When $\eta_g \to 0$, the 2*D* DR becomes

$$\frac{k_x^2}{[\overline{a}_0(E)]^2} + \frac{k_y^2}{[\overline{b}_0(E)]^2} + \frac{1}{[\overline{c}_0(E)]^2}(n_z\pi/d_z)^2 = 1 \tag{3.37}$$

From Eq. (3.37), the EEM and n_0 become [122]

$$m^*(E_{F_s}, n_z) = \frac{\hbar^2}{2}[P^2(E_{F_s}, n_z)\overline{a}_0(E_{F_s})\overline{b}_0(E_{F_s})]' \tag{3.38}$$

$$n_0 = z_2 \sum_{n_z=1}^{n_{z\max}} [T_{61}(E_{Fs}, n_z)]. \tag{3.39}$$

3.2.6 The Te

The 2*D* DR in HD QW of Te becomes

$$k_s^2 = \psi_{5\mathrm{HD}}(E,\eta_g) - \psi_6\left(\frac{\pi n_z}{d_z}\right)^2 \pm \psi_7\left[\psi_{8\mathrm{HD}}^2(E,\eta_g) - \left(\frac{\pi n_z}{d_z}\right)^2\right]^{1/2} \tag{3.40}$$

where, $\psi_7 = \frac{\psi_4\sqrt{\psi_1}}{\psi_2^{3/2}}$ and $\psi_{8\mathrm{HD}}^2(E,\eta_g) = \left[\frac{\psi_4+4\gamma_3(E,\eta_g)\psi_2\psi_4^2+4\psi_2^2\psi_3^2}{4\psi_1\psi_2\psi_4^2}\right]$.

The EEM and n_0 become [122]

$$m^*(E_{F1\mathrm{HD}},\eta_g,n_z) = \frac{\hbar^2}{2}\left[\psi'_{5\mathrm{HD}}(E_{F1\mathrm{HD}},\eta_g) \pm \frac{\psi_{8\mathrm{HD}}(E_{F1\mathrm{HD}},\eta_g)\psi'_{8\mathrm{HD}}(E_{F1\mathrm{HD}},\eta_g)}{\sqrt{\psi_{8\mathrm{HD}}^2(E_{F1\mathrm{HD}},\eta_g) - (\pi n_z/d_z)^2}}\right] \tag{3.41}$$

$$n_0 = \frac{g_v}{\pi}\sum_{n_z=1}^{n_{z\max}}[t_{1\mathrm{HDTE}}(E_{F1\mathrm{HD}},\eta_g,n_z)] \tag{3.42}$$

where, $t_{1\mathrm{HDTE}}(E_{F1\mathrm{HD}},\eta_g,n_z) = \left[\psi_{5\mathrm{HD}}(E_{F1\mathrm{HD}},\eta_g,n_z) - \psi_6\left(\frac{\pi n_z}{d_z}\right)^2\right]$.

When $\eta_g \to 0$, the Eq. (3.40) becomes

$$k_s^2 = \psi_5(E) - \psi_6\left(\frac{\pi n_z}{d_z}\right)^2 \pm \psi_7\left[\psi_8^2(E) - \left(\frac{\pi n_z}{d_z}\right)^2\right]^{1/2} \tag{3.43}$$

where, $\psi_5(E) = \left[\frac{E}{\psi_2} + \frac{\psi_4^2}{2\psi_2^2}\right]$ and $\psi_8^2(E) = \frac{\psi_4^4+4E\psi_2\psi_4^2+4\psi_2^2\psi_3^2}{4\psi_1\psi_2\psi_4^2}$

Using Eq. (3.43), the EEM and n_0 become [122]

$$m^*(E_{F_s},n_z) = \frac{\hbar^2}{2}t'_{40}(E_{F_s},n_z) \tag{3.44}$$

$$n_0 = 2z_2\sum_{n_z=1}^{n_{\max}}[t_{40}(E_{Fs},n_z)]. \tag{3.45}$$

3.2.7 The Gallium Phosphide

The DR in HD n-GaP assumes the form

$$\gamma_3(E,\eta_g)=\frac{\hbar^2k_s^2}{2m_\perp^*}+\frac{\hbar^2}{2m_{||}^*}[\overline{A}'k_s^2+k_z^2]-\left[\frac{\hbar^4k_0^2}{m_{||}^{*2}}(k_s^2+k_z^2)+|V_G|^2\right]^{1/2}-|V_G| \tag{3.46}$$

The EEMs become

$$\begin{aligned}m_z^*(E_{F_h},\eta_g)=&\frac{\hbar^2\gamma_3'(E_{F_h},\eta_g)}{b}\\&[1\pm(C+bD)[C^2+4bD^2+4bC\gamma_3(E_{F_h},\eta_g)\\&-4bCD+4b^2\gamma_3(E_{F_h},\eta_g)D]^{-1/2}]\end{aligned} \tag{3.47}$$

and

$$m_s^*(E_{F_h},\eta_g)=\frac{\hbar^2}{2}\left[t_{11}\gamma_3'(E_{F_h},\eta_g)-t_{41}t_5'(E_{F_h},\eta_g)\right] \tag{3.48}$$

The n_0 becomes

$$n_0=\frac{g_v}{4\pi^2}[\overline{I}_{127}(E_{F_h},\eta_g)] \tag{3.49}$$

The $2D$ DR in QW of HD GaP is given by [6]

$$k_s^2=t_{11}\gamma_3(E,\eta_g)+t_{21}-t_{31}\left(\frac{\pi n_z}{d_z}\right)^2-t_{41}\left[\left(\frac{\pi n_z}{d_z}\right)^2+t_5^2(E,\eta_g)\right]^{1/2} \tag{3.50}$$

The EEM and n_0 become

$$\begin{aligned}m^*(E_{F1\mathrm{HD}},\eta_g,n_z)=&\frac{\hbar^2}{2}[t_{11}\gamma_3'(E_{F1\mathrm{HD}},\eta_g)\\&-t_{41}t_5(E_{F1\mathrm{HD}},\eta_g)t_5'(E_{F1\mathrm{HD}},\eta_g)\\&\left[\left(\frac{\pi n_z}{d_z}\right)^2+t_5^2(E_{F1\mathrm{HD}},\eta_g)\right]^{-1/2}\end{aligned} \tag{3.51}$$

$$n_0=\frac{g_v}{\pi}\sum_{n_z=1}^{n_{\max}}[t_{3\mathrm{HDGaP}}(E_{F1\mathrm{HD}},\eta_g,n_z)] \tag{3.52}$$

where $t_{3\text{HD Gap}}(E_{F1\text{HD}}, \eta_g, n_z) = [t_{11}\gamma_3(E_{F1\text{HD}}, \eta_g) + t_{21}\left(\frac{\pi n_z}{d_z}\right)^2 - t_{41}\left[\left(\frac{\pi n_z}{d_z}\right)^2 + t_5^2(E_{F1\text{HD}}, \eta_g)\right]^{1/2}$

The 2*D* DR when $\eta_g \to 0$ becomes

$$k_s^2 = t_{42}(E, n_z) \tag{3.53}$$

Using Eq. (3.53), the EEM becomes

$$m^*(E_{F_s}, n_z) = \frac{\hbar^2}{2} t'_{42}(E_{F_s}, n_z) \tag{3.54}$$

Here, n_0 becomes

$$n_0 = \left[\left(\frac{g_v}{4\pi a^2}\right) \sum_{n_z=1}^{n_{z\max}} [t_{42}(E_{Fs}, n_z)\right]. \tag{3.55}$$

3.2.8 The Platinum Antimonide

Here, the 2*D* HDDR becomes

$$T_{11}k_s^4 - k_s^2[T_{21}(E, \eta_g) - T_{31}k_z^2] + [T_{41}k_z^4 - T_{51}(E, \eta_g)k_z^2 - T_{61}(E, \eta_g)] = 0 \tag{3.56}$$

The EEMs are given by

$$m_s^*(E_{F_h}, \eta_g) = \frac{\hbar^2}{2T_{11}}[T'_{21}(E_{F_h}, \eta_g) + \frac{(T_{21}(E_{F_h}, \eta_g)T'_{21}(E_{F_h}, \eta_g) + 2T_{11}T'_{61}(E_{F_h}, \eta_g)}{\sqrt{T_{21}^2(E_{F_h}, \eta_g) + 4T_{11}T_{61}(E_{F_h}, \eta_g)}} \tag{3.57}$$

$$\begin{aligned} m_z^*(E_{F_h}, \eta_g) = \left(\frac{\hbar^2}{2T_{41}}\right)&[T'_{51}(E_{F_h}, \eta_g) \\ &+ [T'_{51}(E_{F_h}, \eta_g)T'_{51}(E_{F_h}, \eta_g) + 2T'_{41}T'_{61}(E_{F_h}, \eta_g)] \\ &\times [T_{51}^2(E_{F_h}, \eta_g) + 4T_{41}T_{61}(E_{F_h}, \eta_g)]^{-1/2}] \end{aligned} \tag{3.58}$$

The n_0 assumes the form

$$n_0 = \frac{g_v}{3\pi^2}[\overline{I}_{128}(E_{F_h}, \eta_g)] \tag{3.59}$$

From Eq. (3.60), we can write

$$k_s^2 = A_{60}(E, \eta_g, n_z) \tag{3.60}$$

where

$$A_{60}(E, \eta_g, n_z) = [P_{1\mathrm{HD}}(E, \eta_g, n_z) - \sqrt{P_{1\mathrm{HD}}^2(E, \eta_g, n_z) - 4T_{11}P_{2\mathrm{HD}}(E, \eta_g, n_z)}].$$

The EEM and n_0 become

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z) = \frac{\hbar^2}{2}A'_{60}(E_{F1\mathrm{HD}}, \eta_g, n_z) \tag{3.61}$$

$$n_0 = z_2 \sum_{n_z=1}^{n_{z\max}} [A_{60}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.62}$$

The 2*D* DR in QWs of n-$PtSb_2$, when $\eta_g \to 0$ is given by

$$k_s^2 = t_{44}(E, \eta_z) \tag{3.63}$$

Using (3.63), the EEM and n_0 become

$$m^*(E_{F_s}, n_z) = \frac{\hbar^2}{2}t'_{44}(E_{F_s}, n_z) \tag{3.64}$$

$$n_0 = z_2 \sum_{r=1}^{n_{z\max}} [t_{44}(E_{Fs}, n_z)]. \tag{3.65}$$

3.2.9 The Bismuth Telluride

Here, the 2*D* HDDR becomes

$$\gamma_2(E, \eta_g) = \overline{\omega}_1\left(\frac{\pi n_x}{d_x}\right)^2 + \overline{\omega}_2 k_y^2 + \overline{\omega}_3 k_z^2 + 2\overline{\omega}_4 k_z k_y \tag{3.66}$$

The EEM and n_0 become

$$m^*(E_{F1\mathrm{HD}}, \eta_g) = \frac{m_0}{\sqrt{\alpha_1\alpha_{33} - 4\alpha_{23}^2}}\gamma_2'(E_{F1\mathrm{HD}}, \eta_g) \tag{3.67}$$

$$n_0 = z_2[\sum_{n_z=1}^{n_{z\max}} R_{60}(E_{F1\mathrm{HD}}, \eta_g, n_x)] \tag{3.68}$$

where $R_{60}(E_{F1\mathrm{HD}}, \eta_g, n_z) = \frac{1}{\sqrt{a_{11}\alpha_{33} - 4\alpha_{23}^2}}\left[\frac{2m_0\gamma_2'(E_{F1\mathrm{HD}}, \eta_g)}{\hbar^2} - \frac{2m_0}{\hbar^2}\left(\frac{\pi n_x}{d_x}\right)^2\overline{\alpha}_{11}\right]$

The $2D$ DR in QWs of Bi_2Te_3 when $\eta_g \to 0$ assumes the form

$$E(1 + \alpha E) = \overline{\omega}_1\left(\frac{n_x\pi}{d_x}\right)^2 + \overline{\omega}_2k_y^2 + \overline{\omega}_3k_z^2 + 2\overline{\omega}_4k_zk_y \tag{3.69}$$

The EEM and n_0 become

$$m^*(E_{F_s}) = \frac{m_0(1 + 2\alpha(E_{F_s}))}{\sqrt{\overline{\alpha}_{22}\overline{\alpha}_{33} - 4\overline{\alpha}_{23}^2}}$$

$$n_0 = \frac{g_v}{2\pi}\left(\frac{m_0z_2}{\sqrt{\overline{\alpha}_{22}\overline{\alpha}_{33} - 4\alpha_{23}^2}}\right)\sum_{n_z=1}^{n_{z\max}}\left[\frac{2m_0E_{F_s}(1 + \alpha E_{F_s})}{\hbar^2} - \overline{\omega}_1\left(\frac{n_x\pi}{d_x}\right)^2\right] \tag{3.70}$$

3.2.10 The Germanium

(a) **The Model Cardona et al.**

Here, the $2D$ HDDR becomes

$$\frac{\hbar^2k_s^2}{2m_\perp^*} = \gamma_2(E, \eta_g) + \alpha\left(\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_\parallel^*}\right) - (1 + 2\alpha\gamma_3(E, \eta_g))\left(\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_\parallel^*}\right) \tag{3.71}$$

The n_0 can be written as

$$n_0 = \frac{g_vm_\perp^*}{\pi\hbar^2}\left[\sum_{n_z=1}^{n_{z\max}} R_1(E_{F1\mathrm{HD}}, \eta_g, n_x)\right] \tag{3.72}$$

where, $R_1(E_{F1HD}, \eta_g, n_z) = [\gamma_2(E_{F1HD}, \eta_g) + \alpha(\frac{\hbar^2(\frac{n_z\pi}{d_z})^2}{2m_\parallel^*}) - (1 + 2\alpha\gamma_3(E_{F1HD}, \eta_g))(\frac{\hbar^2(\frac{n_z\pi}{d_z})^2}{2m_\parallel^*})]$

The 2*D* DR in QWs of n-Ge when $\eta_g \to 0$ becomes [122]

$$\frac{\hbar^2 k_x^2}{2m_1^*} + \frac{\hbar^2 k_y^2}{2m_2^*} = \gamma(E, n_z) \tag{3.73}$$

The EEM and n_0 become

$$m^*(E_{F_s}, n_z) = (\sqrt{m_1^* m_2^*})[\gamma(E_{F_s}, n_z)]' \tag{3.74}$$

$$n_0 = \frac{4\sqrt{m_1^* m_2^*}}{\pi\hbar^2} \times \sum_{n_z=1}^{n_{z\max}} [\gamma(E_{F_s}, n_z)] \tag{3.75}$$

(b) **Model of Wang and Ressler et al.**

The HDDR becomes [122]

$$\frac{\hbar^2 k_s^2}{2m_\perp^*} = \overline{\alpha}_8 - \overline{\alpha}_9 \mathrm{k}_z^2 - \overline{\alpha}_{10}[\mathrm{k}_z^4 + \overline{\alpha}_{11}\mathrm{k}_z^2 + \overline{\alpha}_{12}(\mathrm{E}, \eta_g)]^{1/2} \tag{3.76}$$

The EEM's and n_0 become

$$m_z^*(E_{Fh}, \eta_g) = \left[\frac{m_{||}^* \gamma_3'(E_{Fh}, \eta_g)}{\sqrt{1 - 4\overline{\alpha}_6 \gamma_3(E_{Fh}, \eta_g)}}\right] \tag{3.77}$$

$$m_\perp^*(E_{Fh}, \eta_g) = \left[\frac{m_\perp^* \gamma_3'(E_{Fh}, \eta_g)}{\sqrt{1 - 4\overline{\alpha}_4 \gamma_3(E_{Fh}, \eta_g)}}\right] \tag{3.78}$$

$$n_0 = \frac{m_\perp^* g_v}{\pi^2\hbar^2}[I_3(E_{F_h}, \eta_s)] \tag{3.79}$$

Here, the 2*D* DR in QW of HD Ge becomes

$$\frac{\hbar^2 k_s^2}{2m_\perp^*} = A_{75}(E, \eta_g, n_z) \tag{3.80}$$

The EEM and n_0 become

$$m_s(E_{F1\mathrm{HD}}, \eta_g, n_z) = m_\perp^* A_{75}'(E_{F1\mathrm{HD}}, \eta_g, n_z) \tag{3.81}$$

$$n_0 = \frac{m_\perp^* g_v}{\pi\hbar^2} \sum_{n_z=1}^{n_{z\max}} [A_{75}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.82}$$

The 2*D* DR when $\eta_g \to 0$ becomes

$$\frac{\hbar^2 k_x^2}{2m_1^*} + \frac{\hbar^2 k_y^2}{2m_2^*} = I_1(E, n_z) \tag{3.83}$$

where, $I_1(E, n_z) = (2\overline{\alpha}_4)^{-1}[A_6(n_z) - 4\overline{\alpha}_4 E + 4\overline{\alpha}_4 A_5(n_z)]^{1/2}]$

Using (3.87), the EEM and n_0 become

$$m^*(E_{F_s}, n_z) = (\sqrt{m_1^* m_2^*})[I_1(E_{F_s}, n_z)]' \tag{3.84}$$

$$n_0 = \frac{4\sqrt{m_1^* m_2^*}}{\pi\hbar^2} \sum_{n_z=1}^{n_{z\max}} [I_1(E_{F_s}, \eta_z)]. \tag{3.85}$$

3.2.11 The Gallium Antimonide

Here, the HDDR becomes [155]

$$\frac{\hbar^2 k^2}{2m_c} = I_{36}(E, \eta_g) \tag{3.86}$$

The EEM and n_0 become

$$m^*(E_{F_h}, \eta_g) = m_c\{I_{36}(E_{F_h}, \eta_g)\}' \tag{3.87}$$

$$n_0 = \frac{g_v}{3\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} [\{I_{36}(E_{F_h}, \eta_g)\}^{3/2}] \tag{3.88}$$

The 2*D* HDDR becomes

$$\overline{G}_4 k_s^2 + \overline{G}_5 = I_{36}(E, \eta_g) \tag{3.89}$$

The EEM and n_0 become

$$m^*(E_{F1\mathrm{HD}}, \eta_g, n_z) = m_c[I'_{36}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.90}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{119D}(E_{F1\mathrm{HD}}, \eta_g, n_z)] \tag{3.91}$$

The 2*D* EEM and n_0 under the condition $\eta_g \to 0$ become

$$m^*(E_{F_s}) = (m_c)[I_{36}(E_{F_s})]' \tag{3.92}$$

$$n_0 = \overline{G}_6 \sum_{n_z=1}^{n_{z\max}} [T_{55}(E_{Fs}, n_z)]. \tag{3.93}$$

3.3 Result and Discussions

From the Figs. 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 and 3.7, the following points can be written:

1. From Fig. 3.1, we note that Ω_1 is a monotonic increasing function with increasing n_0.
2. From Fig. 3.2, we note that Ω_1 is a monotonic increasing function with decreasing n_0.
3. From Fig. 3.3 illustrates that Ω_1 is a decreasing function with increasing n_0 in distorted periodic ways.
4. From Figs. 3.4 and 3.5, we note that Ω_1 is a decreasing function with increasing d_y in periodic ways.
5. From Fig. 3.6, we note that Ω_1 increases with increasing n_0 for the indicated materials.
6. From Fig. 3.7, we note that Ω_1 increases with decreasing x in different ways.

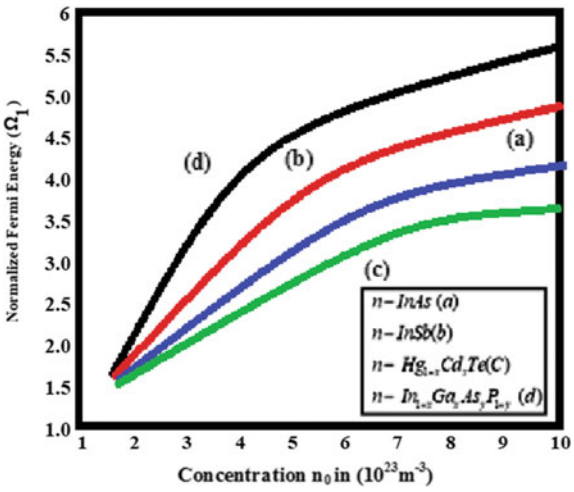

Fig. 3.1 The Ω_1 versus n_0 for the indicated HD compounds

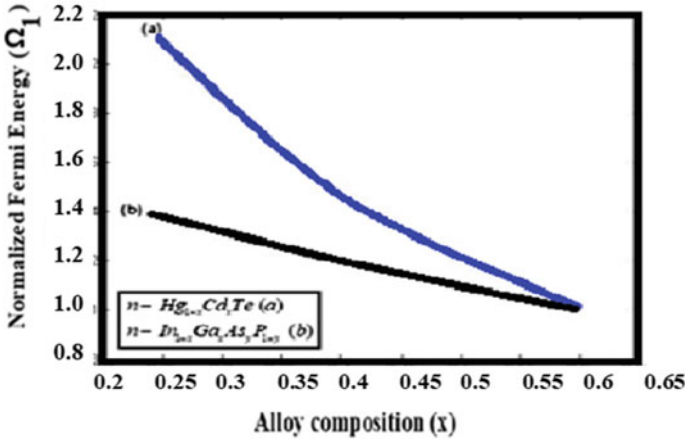

Fig. 3.2 The Ω_1 versus n_0 for the indicated HD compounds

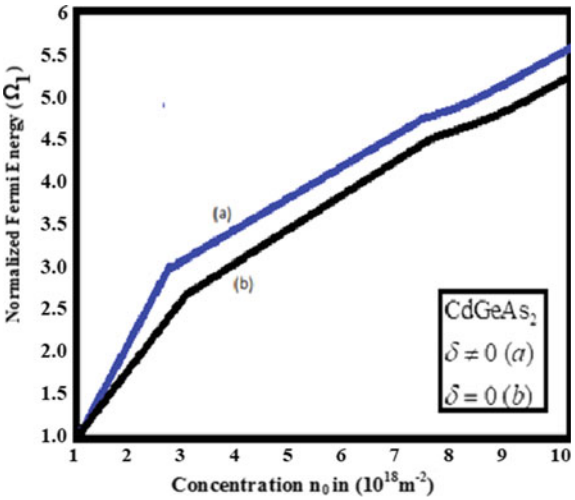

Fig. 3.3 The Ω_1 versus n_0 for QWs of the indicated HD material (10^{18} m^{-2})

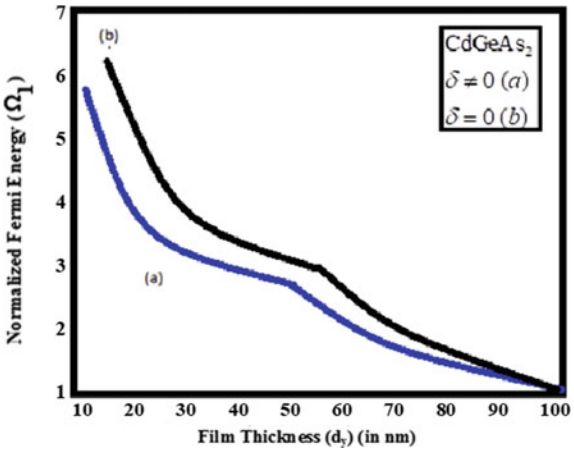

Fig. 3.4 The Ω_1 versus d_y for QWs of the indicated HD material

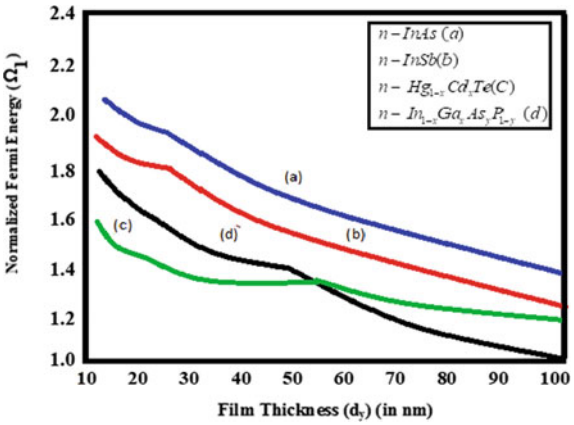

Fig. 3.5 The Ω_1 versus d_y for QWs of the indicated HD material

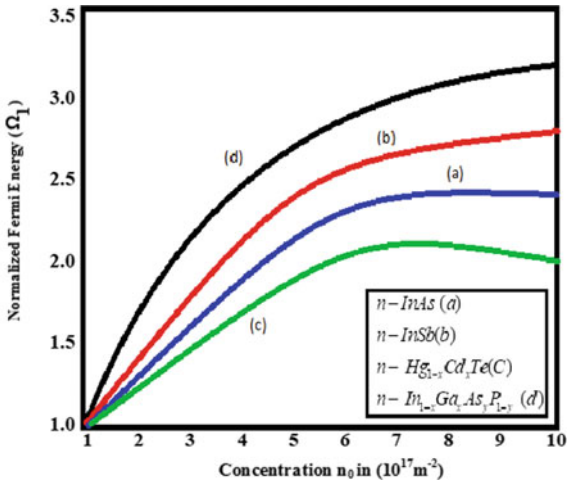

Fig. 3.6 The Ω_1 versus n_0 for QWs of the indicated HD material

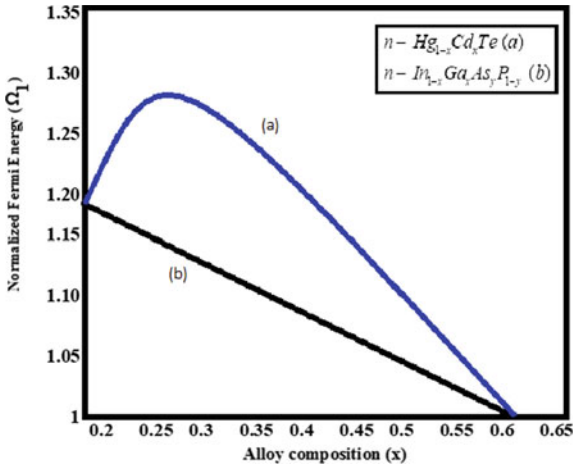

Fig. 3.7 The Ω_1 versus x for QWs of the indicated HD material

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Chapter 4
The HUP and the ES in Nanowires of HD Non-parabolic Materials

4.1 Introduction

In nanowires (NWs), the carriers move in one direction continuously since the other two directions are being quantized and they have extensive device applications in low dimensional electronics [1–27]. In this chapter, we study chronologically the n_0 and EEM for the HDNWs as written in the content of the said chapter. The Sect. 4.3 contains the result and discussions.

4.2 Mathematical Basis

4.2.1 The Nonlinear Optical Materials

Here, the $1D$ DR becomes [28]

$$k_x = T_{1\mathrm{HDNW}}(\Delta_9, \eta_g) \tag{4.1}$$

where, $\Delta_9 = (E, n_y, n_z)$ and $T_{1\mathrm{HDNW}}(\Delta_9, \eta_g) = [[1 - [\frac{\hbar^2(n_z\pi/d_z)^2}{2m^*_{\|}T_{21}(E,\eta_g)} + \frac{\hbar^2(n_y\pi/d_y)^2}{2m^*_{\perp}T_{22}(E,\eta_g)}]]\frac{2m^*_{\perp}T_{21}(E,\eta_g)}{\hbar^2}]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\left[R.P.\frac{\partial}{\partial(E_{F1\mathrm{HDNW}})}[T_{1\mathrm{HDNW}}(\Delta_9, \eta_g)]^2\right] \tag{4.2}$$

$$n_{01D} = R.P.\bar{n}_{01D} \tag{4.3}$$

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where, $\Delta_{10} = (E_{F1\text{HDNW}}, n_y, n_z)$, $E_{F1\text{HDNW}}$ is the corresponding Fermi energy, and $\overline{n}_{01D} = \theta_2[T_{1\text{HDNW}}(\Delta_{10}, \eta_g)$

in which $\theta_2 = \frac{2g_v}{\pi} \sum_{n_y=1}^{n_{y\max}} \sum_{n_z=1}^{n_{z\max}}$.

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = f_{10}(\Delta_9) \tag{4.4}$$

where $f_{10}(\Delta_9) = [f_1(E)]^{-1/2}\left[\gamma(E) - f_1(E)\left(\frac{n_y\pi}{d_y}\right)^2 - f_2(E)\left(\frac{n_z\pi}{d_z}\right)^2\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2} f'_{10}(\Delta_{11}) \tag{4.5}$$

$$n_{01D} = \theta_2[[f_{10}(\Delta_{11})]] \tag{4.6}$$

where $\Delta_{11} = (E_{F1d}, n_y, n_z)$ and E_{F1d} is the corresponding Fermi energy.

4.2.2 The III-V Materials

(i) **Three-Band Model of Kane**

Here, the $1D$ DR becomes

$$k_x = T_2(\Delta_9, \eta_g) \tag{4.7}$$

$T_2(\Delta_9, \eta_g) \equiv [[T_{31}(E, \eta_g) + iT_{32}(E, \eta_g) - [G_2(\Delta_{12})]]\frac{2m_c}{\hbar^2}]^{1/2}$, $\Delta_{12} = (n_y, n_z)$ and $G_2(n_y, n_z) = \left(\frac{\hbar^2\pi^2}{2m}\right)\left[\left(\frac{n_z}{d_z}\right)^2 + \left(\frac{n_y}{d_y}\right)^2\right]$

The EEM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[T'_{31}(E_{F1\text{HDNW}}, \eta_g)] \tag{4.8}$$

$$n_{01D} = R.P\ \overline{n}_{01D} \tag{4.9}$$

where, $\overline{n}_{01D} = \theta_2[T_{2\text{HDNW}}(\Delta_{10}, \eta_g)]$

When $\eta_g \to 0$, $1D$ DR become

$$k_x = f_{11}(\Delta_9) \tag{4.10a}$$

where, $f_{11}(\Delta_9) \equiv \left[\frac{2m_c}{\hbar^2}[I_{11}(E) - G_2(\Delta_{12})]\right]^{1/2}$

The EEM becomes

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[T'_{31}(E_{F1\text{HDNW}}, \eta_g)] \tag{4.10b}$$

The n_{01D} becomes

$$n_{01D} = \theta_2[f_{11}(\Delta_{11})] \tag{4.11}$$

where, E_{F1D} is the corresponding Fermi energy.

(ii) **Two-Band Model of Kane**

Here, the $1D$ HDDR becomes

$$k_x = T_3(\Delta_9, \eta_g) \tag{4.12}$$

where, $T_3(\Delta_9, \eta_g) = \left[[\gamma_2(E, \eta_g) - [G_2(\Delta_{12})]]\frac{2m_c}{\hbar^2}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[\gamma'_2(E_{F1\text{HDNW}}, \eta_g)] \tag{4.13}$$

$$n_{01D} = \theta_2[T_3(\Delta_{10}, \eta_g)] \tag{4.14}$$

The $1D$ DR, EEM and n_{01D} for $\eta_g \to 0$ become

$$k_x = f_{12}(\Delta_9) \tag{4.15}$$

$$m^*(E_{F1D}) = m_c[(1 + 2\alpha E_{F1D})] \tag{4.16}$$

$$n_{01D} = \theta_2[f_{12}(\Delta_{11})] \tag{4.17}$$

where, $f_{12}(\Delta_9) \equiv \left[\frac{2m_c}{\hbar^2}[E(1 + \alpha E) - G_2(\Delta_{12})]\right]^{1/2}$

(iii) **Parabolic Energy Bands**

Here, the $1D$ HDDR becomes

$$k_x = T_4(\Delta_9, \eta_g) \tag{4.18}$$

where, $T_4(\Delta_9, \eta_g) = \left[[\gamma_3(E, \eta_g) - [G_2(\Delta_{12})]]\frac{2m_c}{\hbar^2}\right]^{1/2}$.

The EEM and n_{01D} becomes

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[\gamma_3'(E_{F1\text{HDNW}}, \eta_g)] \tag{4.19}$$

$$n_{01D} = \theta_2[T_4(\Delta_{10}, \eta_g) \tag{4.20}$$

The $1D$ DR, EEM and n_{01D} for $\eta_g \to 0$ become

$$k_x = f_{13}(\Delta_9) \tag{4.21}$$

$$m^*(E_{F1D}) = m_c \tag{4.22}$$

$$n_{01D} = \theta_2[f_{13}(\Delta_{11})] \tag{4.23}$$

where, $f_{13}(\Delta_9) \equiv \left[\frac{2m_c}{\hbar^2}[E - G_2(\Delta_{12})]\right]^{1/2}$

(iv) **The Model of Stillman et al**.

Here, the $1D$ HDDR becomes

$$k_x = T_{11\text{HDNW}}(\Delta_9, \eta_g) \tag{4.24}$$

where, $T_{11\text{HDNW}}(\Delta_9, \eta_g) = \left[[\theta_4(E, \eta_g) - G_2(\Delta_{12})]\frac{2m_c}{\hbar^2}\right]^{1/2}$ and $\theta_4(E, \eta_g) = I_{12}(E, \eta_g)$.

The EM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[\theta_4'(E_{F1\text{HDNW}}, \eta_g)] \tag{4.25}$$

$$n_{01D} = \theta_2[T_{11\text{HDNW}}(\Delta_{10}, \eta_g)] \tag{4.26}$$

where $E_{F1\text{HDNW}}$ is the corresponding Fermi energy.

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = f_{14}(\Delta_9) \tag{4.27}$$

where, $f_{14}(\Delta_9) \equiv \left[\frac{2m_c}{\hbar^2}[I_{12}(E) - G_2(\Delta_{12})]^{1/2}\right]$

The EEM and n_{01D} become

$$m^*(E_{F1D}) = m_c I_{12}'(E_{F1D}) \tag{4.28}$$

$$n_{01D} = \theta_2[f_{14}(\Delta_{11})] \tag{4.29}$$

(v) **The Model of Palik et al.**

Here, the $1D$ HDDR becomes

$$k_x = T_{6\text{HDNW}}(\Delta_9, \eta_g) \tag{4.30}$$

where, $T_{6\text{HDNW}}(\Delta_9, \eta_g) = \left[[\theta_5(E, \eta_g) - G_2(\Delta_{12})]\frac{2m_c}{\hbar^2}\right]^{1/2}$.

The EEM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[\theta_5'(E_{F1\text{HDNW}}, \eta_g)] \tag{4.31}$$

$$n_{01D} = \theta_2[T_{6\text{HDNW}}(\Delta_{10}, \eta_g)] \tag{4.32}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = f_{15}(\Delta_9) \tag{4.33}$$

where $f_{15}(\Delta_9) \equiv \left[\frac{2m_c}{\hbar^2}[I_{13}(E) - G_2(\Delta_{12})]\right]^{1/2}$.

The EEM and n_{01D} become

$$m^*(E_{F1D}) = m_c I_{13}'(E_{F1D}) \tag{4.34}$$

$$n_{01D} = \theta_2[f_{15}(\Delta_{11})]. \tag{4.35}$$

4.2.3 The II-VI Materials

Here, the 1D HDDR becomes

$$k_z = T_7(\Delta_9, \eta_g) \tag{4.36}$$

where, $T_7(\Delta_9, \eta_g) \equiv \left[[\gamma_3(E, \eta_g) - [G_{3,\pm}(\Delta_{13})]]\frac{2m_{||}^*}{\hbar^2}\right]^{1/2}$, $\Delta_{13} = (n_x, n_y)$ and $G_{3,\pm}(\Delta_{13}) = a_0'\left[\left(\frac{n_x\pi}{d_x}\right)^2 + \left(\frac{n_y\pi}{d_y}\right)^2\right] \pm \bar{\lambda}_0\left[\left(\frac{n_x\pi}{d_x}\right)^2 + \left(\frac{n_y\pi}{d_y}\right)^2\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_{||}^*\gamma_3'(E_{F1\text{HDNW}}, \eta_g) \tag{4.37}$$

$$n_{01D} = (\theta_2/2)[T_7(\Delta_{14}, \eta_g) \tag{4.38}$$

where, $\Delta_{14} = (E_{F1\text{HDNW}}, n_x, n_y)$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_z = f_{20}(E, n_x, n_y) \tag{4.39}$$

where $f_{20}(E, n_x, n_y) = \left[\frac{2m_{||}^*}{\hbar^2}[E - G_{3,\pm}(\Delta_{13})]\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(E_{F1D}) = m_{||}^* \tag{4.40}$$

$$n_{01D}(E) = (\theta_2/2)[f_{20}(\Delta_{15})] \tag{4.41}$$

where $\Delta_{15} = (E_{F1D}, n_x, n_y)$.

4.2.4 The IV-VI Materials

(i) **Dimmock Model**

Here, the $1D$ HDDR becomes

$$k_z = T_{36}(\Delta_9, \eta_g) \tag{4.42}$$

The EEM and n_{01D} become

$$m^*(\Delta_{14}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[T_{36}^2(\Delta_{14}, \eta_g)] \tag{4.43}$$

$$n_{01D} = \theta_2[T_{36\text{HDNW}}(\Delta_{14}, \eta_g)] \tag{4.44}$$

Here, the $1D$ HDDR becomes

$$k_z = T_{40}(\Delta_9) \tag{4.45}$$

The EEM and n_{01D} become

$$m^*(\Delta_{15}) = \frac{\hbar^2}{2}\frac{\partial}{\partial \overline{E}}[T_{40}^2(\Delta_{15})] \tag{4.46}$$

$$n_{01D} = \theta_2[T_{40}(\Delta_{15})] \tag{4.47}$$

(ii) **Bangert and Kastner Model**

Here, the $1D$ HDDR becomes

$$k_z = T_{60}(\Delta_{16}, \eta_g) \tag{4.48}$$

where $\Delta_{16} = (E, n_x, n_y)$ and $T_{60}(\Delta_{16}, \eta_g) = \left[\left[1 - F_1(E, \eta_g)\left[\left(\frac{n_x\pi}{d_x}\right)^2 + \left(\frac{n_y\pi}{d_y}\right)^2\right]\right][F_2(E, \eta_g)]^{-1}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{14}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial \overline{E}}[T_{60}^2(\Delta_{14}, \eta_g)] \tag{4.49}$$

$$n_{01D} = R.P.\overline{n}_{01D} \tag{4.50}$$

where, $\overline{n}_{01D} = \theta_2[T_{40}(\Delta_{14}\eta_g)]$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_z = T_{61}(\Delta_{16}) \tag{4.51}$$

where, $T_{61}(\Delta_{16}) = \left[\left[1 - \omega_1(E)\left[\left(\frac{n_x\pi}{d_x}\right)^2 + \left(\frac{n_y\pi}{d_y}\right)^2\right]\right][\omega_2(E)]^{-1}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{15}) = \frac{\hbar^2}{2}\frac{\partial}{\partial \overline{E}}[T_{61}^2(\Delta_{15})] \tag{4.52}$$

$$n_{01D} = \theta_2[T_{61}(\Delta_{15})]. \tag{4.53}$$

4.2.5 The Stressed Kane-Type Materials

Here, the $1D$ HDDR becomes

$$k_z = T_{70}(\Delta_{16}, \eta_g) \tag{4.54}$$

where $T_{70}(\Delta_{16}, \eta_g) = [[1 - P_{11}(E, \eta_g)(\frac{\pi n_x}{d_x})^2 + Q_{11}(E, \eta_g)(\frac{\pi n_y}{d_y})^2][S_{11}(E, \eta_g)]^{-1}]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{16}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[T_{70}^2(\Delta_{16}, \eta_g)] \tag{4.55}$$

$$n_{01D} = \theta_2[T_{70}(\Delta_{16}\eta_g)] \tag{4.56}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_z = t_{701}(\Delta_{16}) \tag{4.57}$$

where $t_{701}(\Delta_{16}) = \left[\overline{c}_0(E)\left[1 - \left(\frac{\pi n_x}{d_x\overline{a}_0(E)}\right)^2 - \left(\frac{\pi n_y}{d_y\overline{b}_0(E)}\right)^2\right]\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{15}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[t_{701}^2(\Delta_{15})] \tag{4.58}$$

$$n_{01D} = \theta_2[T_{701}(\Delta_{15})]. \tag{4.59}$$

4.2.6 The Te

Here, the 1D HDDR becomes

$$k_x = t_{72}(\Delta_9, \eta_g) \tag{4.60}$$

where $t_{72}(\Delta_9, \eta_g) = [-(\frac{n_y\pi}{d_y})^2 + \psi_{5\text{HD}}(E, \eta_g) - \psi_6(\frac{\pi n_z}{d_z})^2 \pm \psi_7[\psi_{8\text{HD}}^2(E, \eta_g) - (\frac{\pi n_z}{d_z})^2]^{1/2}]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[t_{72}^2(\Delta_{10}, \eta_g)] \tag{4.61}$$

$$n_{01D} = (\theta_2/2)[t_{72}(\Delta_{10}, \eta_g)] \tag{4.62}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = H_{70}(\Delta_9) \tag{4.63}$$

where, $H_{70}(\Delta_9) = \left[-\left(\frac{n_y\pi}{d_y}\right)^2 + \psi_5(E) - \psi_6\left(\frac{\pi n_z}{d_z}\right)^2 \pm \psi_7\left[\psi_8^2(E) - \left(\frac{\pi n_z}{d_z}\right)^2\right]^{1/2}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[H_{70}^2(\Delta_{11})] \tag{4.64}$$

$$n_{01D} = (\theta_2/2)[H_{70}(\Delta_{11})]. \tag{4.65a}$$

4.2.7 The Gallium Phosphide

Here, the $1D$ HDDR becomes

$$k_x = u_{70}(\Delta_9, \eta_g) \tag{4.65b}$$

where $u_{70}(\Delta_9, \eta_g) = [-(\frac{n_y\pi}{d_y})^2 + t_{11}\gamma_3(E, \eta_g) + t_{21} - t_{31}(\frac{n_z\pi}{d_z})^2 - t_{41}[(\frac{n_z\pi}{d_z})^2 + t_5^2(E, \eta_g)]^{1/2}]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[u_{70}^2(\Delta_{10}, \eta_g)] \tag{4.66}$$

$$n_{01D} = (\theta_2)[u_{70}(\Delta_{10}, \eta_g)] \tag{4.67}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = X_{71}(\Delta_9) \tag{4.68}$$

where $X_{71}(\Delta_9) = \left[-\left(\frac{n_y\pi}{d_y}\right)^2 + t_{42}(E, n_z)\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[X_{71}^2(\Delta_{11})] \tag{4.69}$$

$$n_{01D} = (\theta_2)[X_{71}(\Delta_{11})]. \tag{4.70}$$

4.2.8 The Platinum Antimonide

Here, the 1D HDDR becomes

$$k_x = V_{70}(\Delta_9, \eta_g) \tag{4.71}$$

where $V_{70}(\Delta_9, \eta_g) = \left[-\left(\frac{n_y\pi}{d_y}\right)^2 + A_{60}(E, \eta_g, n_z)\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[V_{70}^2(\Delta_{10}, \eta_g)] \tag{4.72}$$

$$n_{01D} = (\theta_2)[V_{70}(\Delta_{10}, \eta_g)] \tag{4.73}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = D_{71}(\Delta_9) \tag{4.74}$$

where $D_{71}(\Delta_9) = \left[-\left(\frac{n_y\pi}{d_y}\right)^2 + t_{44}(E, n_z)\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[D_{71}^2(\Delta_{11})] \tag{4.75}$$

$$n_{01D} = (\theta_2)[D_{71}(\Delta_{11})]. \tag{4.76}$$

4.2.9 The Bismuth Telluride

The 1D HDDR becomes

$$k_x = J_{70}(\Delta_9, \eta_g) \tag{4.77}$$

where $J_{70}(\Delta_9, \eta_g) = \left[\left[\gamma_2(E, \eta_g) - \omega_2\left(\frac{n_y\pi}{d_y}\right)^2 - \omega_3\left(\frac{n_z\pi}{d_z}\right)^2 - 2\omega_4\frac{n_y n_z \pi^2}{d_y d_z}\right](\omega_1)^{-1}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[J_{70}^2(\Delta_{10}, \eta_g) \tag{4.78}$$

$$n_{01D} = (\theta_2)[J_{70}(\Delta_{10}, \eta_g)] \tag{4.79}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = B_{71}(\Delta_9) \tag{4.80}$$

where $B_{71}(\Delta_9) = \left[\left[E(1+\alpha E) - \omega_2\left(\frac{n_y\pi}{d_y}\right)^2 - \omega_3\left(\frac{n_z\pi}{d_z}\right)^2 - 2\omega_4\frac{n_y n_z \pi^2}{d_y d_z}\right](\omega_1)^{-1}\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[B_{71}^2(\Delta_{11})] \tag{4.81}$$

$$n_{01D} = (\theta_2)[B_{71}(\Delta_{11})]. \tag{4.82}$$

4.2.10 The Germanium

(a) **Model of Cardona et al.**

Here, the $1D$ HDDR becomes

$$k_x = L_{70}(\Delta_9, \eta_g) \tag{4.83}$$

where $L_{70}(\Delta_9, \eta_g) = [[\gamma_2(E, \eta_g) + \alpha[\frac{\hbar^2}{2m_{||}^*}(\frac{n_z\pi}{d_z})^2]^2 - (1 + 2\alpha\gamma_3(E, \eta_g))\frac{\hbar^2}{2m_{||}^*}(\frac{n_z\pi}{d_z})^2](\frac{2m_{||}^*}{\hbar^2})]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[L_{70}^2(\Delta_{10}, \eta_g)] \tag{4.84}$$

$$n_{01D} = (\theta_2)[L_{70\text{HDNW}}(\Delta_{10}, \eta_g)] \tag{4.85}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = B_{77}(E, n_y, n_z) \tag{4.86}$$

where $B_{77}(E, n_y, n_z) = [[E(1 + \alpha E) + \alpha[\frac{\hbar^2}{2m_{||}^*}(\frac{n_z\pi}{d_z})^2]^2 - (1 + 2\alpha E)\frac{\hbar^2}{2m_{||}^*}(\frac{n_z\pi}{d_z})^2](\frac{2m_{||}^*}{\hbar^2})]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[B_{77}^2(\Delta_{11})] \tag{4.87}$$

$$n_{1D} = (\theta_2)[B_{77}(\Delta_{11})] \tag{4.88}$$

(b) **Model of Wang et al.**

Here, the 1D HDDR becomes

$$k_x = \beta_{70}(\Delta_9, \eta_g) \tag{4.89}$$

where $\beta_{70}(\Delta_9, \eta_g) = [-(\frac{n_y\pi}{d_y})^2 + \frac{2m_\perp^*}{\hbar^2}[\alpha_8 - \alpha_9(\frac{\pi n_z}{d_z})^2 - \alpha_{10}[(\frac{\pi n_z}{d_z})^4 + \alpha_{11}(\frac{\pi n_z}{d_z})^2 + \alpha_{12}(E, \eta_g)]^{1/2}]]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{10}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[\beta_{70}^2(\Delta_{10}, \eta_g)] \tag{4.90}$$

$$n_{01D} = (\theta_2)[\beta_{70\text{HDNW}}(\Delta_{10}, \eta_g)] \tag{4.91}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = P_{77}(\Delta_9) \tag{4.92}$$

where $P_{77}(\Delta_9) = \left[\left[I_1(E, n_z) - \frac{\hbar^2}{2m_2^*}\left(\frac{n_y\pi}{d_y}\right)^2\right]\left(\frac{2m_1^*}{\hbar^2}\right)\right]^{1/2}$

The EEM and n_{01D} become

$$m^*(\Delta_{11}) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[P_{77}^2(\Delta_{11})] \tag{4.93}$$

$$n_{01D} = (\theta_2)[P_{77}(\Delta_{11})]. \tag{4.94}$$

4.2.11 The Galium Antimonide

Here, the $1D$ HDDR becomes

$$k_x = N_{151}(\Delta_9, \eta_g) \tag{4.95}$$

where, $N_{151}(\Delta_9, \eta_g) = \left[\frac{2m_c}{\hbar^2}[I_{36}(E) - G_2(\Delta_{12})]^{1/2}\right]$

The EEM and n_{01D} become

$$m^*(E_{F1\text{HDNW}}, \eta_g) = m_c[I'_{36}(E_{F1\text{HDNW}}, \eta_g)] \tag{4.96}$$

$$n_{01D} = (\theta_2)[N_{151}(\Delta_{10}, \eta_g)] \tag{4.97}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$k_x = f_{151}(\Delta_9, \eta_g) \tag{4.98}$$

where, $f_{151}(\Delta_9, \eta_g) = \left[\frac{2m_c}{\hbar^2}[I_{36}(E) - G_2(\Delta_{12})]^{1/2}\right]$

The EEM and n_{01D} become

$$m^*(E_{F1D}) = m_c[I'_{36}(E_{F1D})] \tag{4.99}$$

$$n_{01D} = (\theta_2)[f_{151}(\Delta_{11}, \eta_g)]. \tag{4.100}$$

4.2.12 The HD II-V Materials

Here, the $1D$ HDDR becomes

$$k_x = \Delta_{27}(\Delta_{16}, \eta_g) \tag{4.101}$$

where

$$\begin{aligned}
\Delta_{27}(\Delta_{16}, \eta_g) &= [(A_{12}^2 - A_{16}^2)^{-1}[[\Delta_{21}(\Delta_{16}, \eta_g,)A_{12} + A_{16}\Delta_{22}(\Delta_{13})] \\
&\quad + [[\Delta_{21}(\Delta_{16}, \eta_g)A_{12} + A_{16}\Delta_{22}(\Delta_{13})]^2 \\
&\quad - (A_{12}^2 - A_{16}^2)\Delta_{25}(\Delta_{16}, \eta_g)]^{1/2} \\
\Delta_{21}(\Delta_{16}, \eta_g) &= \left[\gamma_3(E, \eta_g) - A_{10}\left(\frac{n_x\pi}{d_x}\right)^2 - A_{11}\left(\frac{n_y\pi}{d_y}\right)^2 - A_{13}\left(\frac{n_x\pi}{d_x}\right)\right], \\
\Delta_{22}(\Delta_{13}) &= \left[A_{14}\left(\frac{n_x\pi}{d_x}\right)^2 + A_{15}\left(\frac{n_y\pi}{d_y}\right)^2 + A_{17}\left(\frac{n_x\pi}{d_x}\right)\right], \\
\Delta_{25}(\Delta_{16}, \eta_g) &= \Delta_{21}^2(\Delta_{16}, \eta_g) - \Delta_{24}(\Delta_{13})], \\
\Delta_{24}(\Delta_{13}) &= [\Delta_{22}^2(\Delta_{16}, \eta_g) + \Delta_{23}(n_y)] \text{ and } \Delta_{23}(n_y) = \left[A_{18}\left(\frac{n_y\pi}{d_y}\right)^2 + A_{19}^2\right]
\end{aligned}$$

The EEM and n_{01D} become

$$m^*(\Delta_{14}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[\Delta_{27}^2(\Delta_{14}, \eta_g)] \tag{4.102}$$

$$n_{1D} = (\theta_2/2)[\Delta_{27}(\Delta_{14}, \eta_g)] \tag{4.103}$$

When $\eta_g \to 0$, $1D$ DR becomes

$$\begin{aligned}
k_z &= \Delta_{271}(\Delta_{16}) \\
&\text{where} \\
\Delta_{271}(\Delta_{16}) &= [(A_{12}^2 - A_{16}^2)^{-1}[[\Delta_{211}(\Delta_{16})A_{12} + A_{16}\Delta_{22}(\Delta_{13})] \\
&\quad + [[\Delta_{211}(\Delta_{16})A_{12} + A_{16}\Delta_{22}(\Delta_{13})]^2 - (A_{12}^2 - A_{16}^2)\Delta_{251}(\Delta_{16})]^{1/2} \\
\Delta_{211}(\Delta_{16}) &= \left[E - A_{10}\left(\frac{n_x\pi}{d_x}\right)^2 - A_{11}\left(\frac{n_y\pi}{d_y}\right)^2 - A_{13}\left(\frac{n_x\pi}{d_x}\right)\right], \\
\Delta_{22}(\Delta_{13}) &= \left[A_{14}\left(\frac{n_x\pi}{d_x}\right)^2 + A_{15}\left(\frac{n_y\pi}{d_y}\right)^2 + A_{17}\left(\frac{n_x\pi}{d_x}\right)\right], \\
\Delta_{251}(\Delta_{16}) &= [\Delta_{211}^2(\Delta_{16}) - \Delta_{24}(\Delta_{13})],\ \Delta_{24}(\Delta_{13}) \\
&= [\Delta_{22}^2(\Delta_{16}) + \Delta_{23}(n_y)] \text{ and } \Delta_{23}(n_y) = \left[A_{18}\left(\frac{n_y\pi}{d_y}\right)^2 + A_{19}^2\right]
\end{aligned} \tag{4.104}$$

The EEM and n_{01D} assume the forms

$$m^*(\Delta_{15}, \eta_g) = \frac{\hbar^2}{2}\frac{\partial}{\partial E}[\Delta_{271}^2(\Delta_{15}, \eta_g)] \tag{4.105}$$

$$n_{01D} = (\theta_2/2)[\Delta_{271}(\Delta_{15}, \eta_g)]. \tag{4.106}$$

4.3 Result and Discussions

From Figs. 4.1, 4.2 and 4.3, we note the following points:

1. From Fig. 4.1, we note that Ω_1 decreases with increasing d_y in an oscillatory way for all the HDNWs of the indicated materials.
2. From Fig. 4.2, we observe that Ω_1 increases with increasing n_{01D} in a periodic manner for all the HDNWs of the indicated materials.

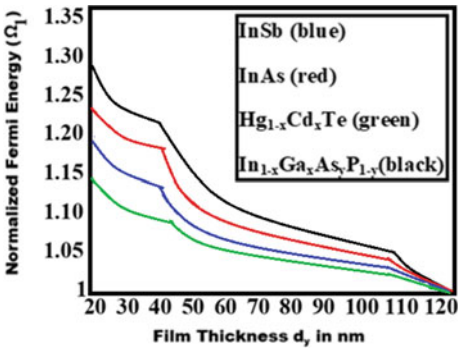

Fig. 4.1 The Ω_1 versus d_y in HD NWs of the indicated materials

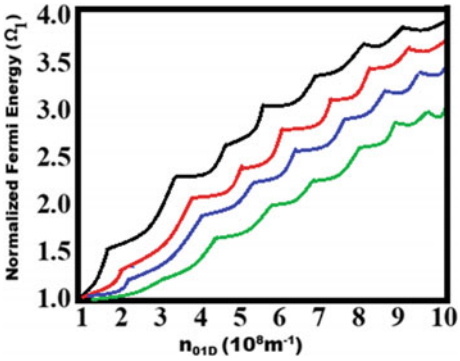

Fig. 4.2 The Ω_1 versus n_{01D} for all cases of Fig. 4.1

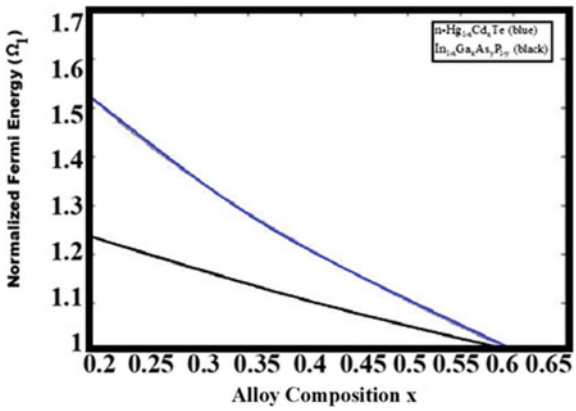

Fig. 4.3 The Ω_1 versus x for the indicated materials

3. From Fig. 4.3, we note that Ω_1 decreases with increasing x in linear fashion for all the HDNWs of the indicated materials.

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Chapter 5
The HUP and the ES in Quantum Dots (QDs) of HD Non-parabolic Materials

5.1 Introduction

In quantum dots (QDs), all the three direction of motions of the carriers are quantized and QDs in recent years are being extensively investigated in the literature for different applications [1–30]. In this chapter, in Sects. 5.2.1, 5.2.2, 5.2.3, 5.2.4, 5.2.5, 5.2.6, 5.2.7, 5.2.8, 5.2.9, 5.2.10, 5.2.11, 5.2.12, we study chronologically the ES in the HD QD materials as written in the content of this chapter. The Sect. 5.3 contains the summary and conclusion.

5.2 Mathematical Basis

5.2.1 The Nonlinear Optical Materials

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{\hbar^2(t_x)^2}{2m^*_{||}T_{21}(E_{1\mathrm{QBHD}},\eta_g)}+\frac{\hbar^2(t_y)^2}{2m^*_{\perp}T_{22}(E_{1\mathrm{QBHD}},\eta_g)}+\frac{\hbar^2(t_z)^2}{2m^*_{\perp}T_{22}(E_{1\mathrm{QBHD}},\eta_g)}=1 \quad (5.1)$$

where, $\overline{t}^i=(\pi n_i/d_i)$

in which $\mathrm{i}=x, y, z$.

The n_0 can be expressed as

$$n_0=\mathrm{R.P.\ of}\ \overline{n}_0 \quad (5.2)$$

where

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$$\overline{n}_0 = \overline{G}_8 F_{-1}(\eta_{1\mathrm{QD}}) \tag{5.3}$$

in which $\eta_{1\mathrm{QD}} = (E_{\mathrm{FQD}} - E_{1\mathrm{QBHD}})(k_B T)^{-1}$

and E_{FQD} is the corresponding Fermi energy.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\gamma(E_{\mathrm{QD1}}) = f_1(E_{\mathrm{QD1}})[(t_x)^2 + (t_y)^2] + f_2(E_{\mathrm{QD1}})[(t_z)^2] \tag{5.4}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{31\mathrm{QD}}) \tag{5.5}$$

where $\eta_{31QD} \equiv (k_B T)^{-1}(E_{F0D} - E_{QD1})$.

5.2.2 The III-V Materials

(a) **The HD Three-Band Model of Kane**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = T_{44}(E_{2\mathrm{QBHD}}, \eta_g) \tag{5.6}$$

$$n_0 = R.P.\,\overline{n}_0 \tag{5.7}$$

where,

$$\overline{n}_0 = \overline{G}_8 F_{-1}(\eta_{2\mathrm{QD}}) \tag{5.8}$$

in which $\eta_{2\mathrm{QD}} = (E_{\mathrm{FQD}} - E_{2\mathrm{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$I_{11}(E_{\mathrm{QD2}}) = \overline{G}_7 \tag{5.9}$$

The n_0 is given by

$$n_0 = \overline{G}_8 F_{-1}(\eta_{32\mathrm{QD}}) \tag{5.10}$$

where $\eta_{32\mathrm{QD}} \equiv (k_B T)^{-1}(E_{F0D} - E_{\mathrm{QD2}})$.

(b) **The HD Two-Band Model of Kane**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = \gamma_2(E_{3\text{QBHD}}, \eta_g) \tag{5.11}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{3\text{QD}}) \tag{5.12}$$

where $\eta_{3\text{QD}} = (E_{\text{FQD}} - E_{3\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$E_{\text{QD3}}(1 + \alpha E_{\text{QD3}}) = \overline{G}_7 \tag{5.13}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{33\text{QD}}) \tag{5.14}$$

where, $\eta_{33\text{QD}} \equiv (k_B T)^{-1}(E_{F0D} - E_{\text{QD3}})$.

(c) **The HD Parabolic Energy Bands**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = \gamma_3(E_{4\text{QBHD}}, \eta_g) \tag{5.15}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{4\text{QD}}) \tag{5.16}$$

where $\eta_{4\text{QD}} = (E_{\text{FQD}} - E_{4\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$E_{\text{QD4}} = \overline{G}_7 \tag{5.17}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{34\text{QD}}) \tag{5.18}$$

where $\eta_{34\text{QD}} \equiv (k_B T)^{-1}(E_{F0D} - E_{\text{QD4}})$.

(d) **The HD Model of Stillman et al.**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = \theta_4(E_{5\text{QBHD}}, \eta_g) \tag{5.19}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{5\text{QD}}) \tag{5.20}$$

where $\eta_{5\text{QD}} = (E_{\text{FQD}} - E_{5\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$I_{12}(E_{\text{QD5}}) = \overline{G}_7 \tag{5.21}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{35\text{QD}}) \tag{5.22}$$

where $\eta_{35\text{QD}} \equiv (k_B T)^{-1}(E_{F0D} - E_{\text{QD5}})$.

(e) **The HD Model of Palik et al.**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = \theta_5(E_{6\text{QBHD}}, \eta_g) \tag{5.23}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{6QD}) \tag{5.24}$$

where $\eta_{6\text{QD}} = (E_{\text{FQD}} - E_{6\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$I_{13}(\overline{E}_{\text{QD6}}) = \overline{G}_7 \tag{5.25}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{36\text{QD}}) \tag{5.26}$$

where, $\eta_{36\text{QD}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{QD6}})$.

(f) **The HD Model of Rossler**

The $0D$ HDDR and n_0 assume the form [30]

$$\begin{aligned}\gamma_3(\overline{E}_{8\text{QBHD}}, \eta_g) \equiv [\overline{G}_7] + \left[\alpha_{11} + \alpha_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right]\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^2 \\ + \left[\beta_{11} + \beta_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right] \\ [(t_x t_y)^2 + (t_y t_z)^2 + (t_z t_x)^2] \pm \left[\gamma_{11} + \gamma_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right] \\ \left[\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right][(t_x t_y)^2 + (t_y t_z)^2 + (t_z t_x)^2] - 9(t_x t_y t_z)^2\right]\end{aligned} \tag{5.27}$$

$$n_0 = \frac{\overline{G}_8 F_{-1}(\overline{\eta}_{8\text{QD}})}{2} \tag{5.28}$$

where $\overline{\eta}_{8\text{QD}} = (E_{\text{FQD}} - \overline{E}_{8\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$(\overline{\overline{E}}_{\mathrm{QD6}_{\pm}}) \equiv \overline{G}_7 + \left[\alpha_{11} + \alpha_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right]\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^2 + \left[\beta_{11} + \beta_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right] [(t_x t_y)^2 + (t_y t_z)^2 + (t_z t_x)^2] \pm \left[\gamma_{11} + \gamma_{12}\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right]^{1/2}\right] \left[\left[\frac{2\overline{G}_7 m_c}{\hbar^2}\right][(t_x t_y)^2 + (t_y t_z)^2 + (t_z t_x)^2] - 9(t_x t_y t_z)^2\right] \tag{5.29}$$

$$n_0 = \frac{\overline{G}_8}{2} F_{-1}(\eta_{37QD}) \tag{5.30}$$

where, $\eta_{37\mathrm{QD}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{\overline{E}}_{\mathrm{QD6}_{\pm}})$.

5.2.3 The II-VI Materials

The $0D$ HDDR and n_0 assume the form [30]

$$\gamma_3(\overline{E}_{9\mathrm{QBHD2}}, \eta_g) = a_0'[(t_x)^2 + (t_y)^2] \pm \overline{\lambda}_0[(t_x)^2 + (t_y)^2]^{1/2} + \frac{\hbar^2 (t_z)^2}{2m_{||}^*} \tag{5.31}$$

$$n_0 = \frac{\overline{G}_8 F_{-1}(\overline{\eta}_{9\mathrm{QD}})}{2} \tag{5.32}$$

where $\overline{\eta}_{9\mathrm{QD}} = (E_{\mathrm{FQD}} - \overline{E}_{9\mathrm{QBHD2}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$E_{\mathrm{QD10},\pm} = a_0'[(t_x)^2 + (t_y)^2] + \frac{1}{2m_{||}^*}(\hbar \bar{t}^z)^2 \pm \overline{\lambda}_0[(t_x)^2 + (t_y)^2]^{1/2} \tag{5.33}$$

$$n_{0D} = \frac{\overline{G}_8 F_{-1}(\eta_{39\mathrm{QD},\pm})}{2} \tag{5.34}$$

where, $\eta_{39QD,\pm} \equiv (k_B T)^{-1}(E_{F0D} - E_{QD10,\pm})$.

5.2.4 The IV-VI Materials

(a) **Dimmock Model**

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_z\pi}{d_z} = T_{36}(E_{81\text{QBHD}}, \eta_g, n_x, n_y) \tag{5.35}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{10\text{QD4}}) \tag{5.36}$$

where $\overline{\eta}_{10\text{QD4}} = (E_{\text{FQD}} - E_{81\text{QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\frac{n_z\pi}{d_z} = T_{40}(E_{QD11}, n_x, n_y) \tag{5.37}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{40QD5}) \tag{5.38}$$

where, $\eta_{40QD5} \equiv (k_B T)^{-1}(E_{F0D} - E_{QD11})$.

(b) **Bangert and Kastner Model**

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_z\pi}{d_z} = T_{60}(E_{9\text{QBHD4}}, \eta_g, n_x, n_y) \tag{5.39}$$

$$n_0 = \text{Real part of}\, \overline{n}_0 \tag{5.40}$$

where,

$$\overline{n}_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{11\text{QD6}}) \tag{5.41}$$

in which $\overline{\eta}_{11\text{QD6}} = (E_{\text{FQD}} - E_{9\text{QBHD4}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\frac{n_z\pi}{d_z} = T_{61}(E_{\text{QD12}}, \eta_g, n_x, n_y) \tag{5.42}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{41\text{QD7}}) \tag{5.43}$$

where, $\eta_{41\text{QD7}} \equiv (k_B T)^{-1}(E_{F0D} - E_{\text{QD12}})$.

(iii) **Model of Foley et al.**

The $0D$ HDDR and n_0 assume the form [30]

$$\gamma_3(\bar{E}_{10\text{QBHD12}}, \eta_g) = -\frac{E_{g_0}}{2} + \frac{\hbar^2}{2m_{\perp}^-}[(t_x)^2 + (t_y)^2] + \frac{\hbar^2}{2m_{||}^-}(\bar{t}^z)^2 + \left[\left[\frac{E_{g_0}}{2} + \frac{\hbar}{2m_{||}^+}(\bar{P}_z)^2 + \frac{\hbar^2}{2m_{\perp}^+}\{(t_x)^2 + (t_y)^2\}\right]^2 + P_{||}^2(t_z)^2 + P_{\perp}^2\{(t_x)^2 + (t_y)^2\}\right]^{1/2} \tag{5.44}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{12\text{QD8}}) \tag{5.45}$$

where $\overline{\eta}_{12\text{QD8}} = (E_{\text{FQD}} - E_{10\text{QBHD12}})(k_B T)^{-1}$.

5.2.5 The Stressed Kane-Type Materials

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_z \pi}{d_z} = T_{70}(E_{10\text{QBHD14}}, \eta_g, n_x, n_y) \tag{5.46}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{13\text{QD10}}) \tag{5.47}$$

where $\overline{\eta}_{13\text{QD10}} = (E_{\text{FQD}} - E_{10\text{QBHD14}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$(t_x)^2[\overline{a}_0(\overline{E}_{\text{QD23}})]^{-2} + (t_y)^2[\overline{b}_0(\overline{E}_{\text{QD23}})]^{-2} + (t_z)^2[\overline{c}_0(\overline{E}_{\text{QD23}})]^{-2} = 1 \tag{5.48}$$

$$n_{0D} = \overline{G}_8 F_{-1}(\eta_{43\text{QD11}}) \tag{5.49}$$

where $\eta_{43\text{QD11}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{QD23}})$.

5.2.6 The Te

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_x\pi}{d_x} = t^i_{72}(E_{11\text{QBHD16}}, n_y, n_z, \eta_g) \tag{5.50}$$

$$n_0 = \frac{\overline{G}_8}{2} F_{-1}(\overline{\eta}_{14QD12}) \tag{5.51}$$

where $\overline{\eta}_{14\text{QD12}} = (E_{\text{FQD}} - E_{11\text{QBHD16}})(k_B T)^{-1}$.

5.2.7 The Gallium Phosphide

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_x\pi}{d_x} = u_{70}(\overline{E}_{14\text{QBHD14}}, n_y, n_z, \eta_g) \tag{5.52}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{15\text{QD14}}) \tag{5.53}$$

where $\overline{\eta}_{15\text{QD14}} = (E_{\text{FQD}} - \overline{E}_{14\text{QBHD14}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\overline{E}_{\text{QD16}} = \frac{\hbar^2}{2m^*_\perp}[(t_x)^2 + (t_y)^2] + \frac{\hbar^2}{2m^*_{||}}[(t_x)^2 + (t_y)^2 + (t_z)^2] - \left[\frac{\hbar^2 k_0^2}{m^{*2}_{||}}\left[(t_x)^2 + (t_y)^2 + (t_z)^2\right] + |V_G|^2\right] + |V_G|^2 \tag{5.54}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{45\text{QD15}}) \tag{5.55}$$

where, $\eta_{45\text{QD15}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{QD16}})$.

5.2.8 The Platinum Antimonide

The $0D$ HDDR and n_0 assume the form [30]

$$\frac{n_x\pi}{d_x} = V_{70}(\overline{E}_{15\text{QBHD15}}, n_y, n_z, \eta_g) \tag{5.56}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{16\text{QD16}}) \tag{5.57}$$

where $\overline{\eta}_{16\text{QD16}} = (E_{\text{FQD}} - \overline{E}_{15\text{QBHD15}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\overline{P}_x = D_{71}(\overline{E}_{\text{QD17}}, n_y, n_z) \tag{5.58}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{46QD17}) \tag{5.59}$$

where, $\eta_{46\text{QD17}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{QD17}})$.

5.2.9 The Bismuth Telluride

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{t}^x = J_{70}(\overline{E}_{\text{18QBHD}}, n_y, n_z, \eta_g) \tag{5.60}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{\text{17QD18}}) \tag{5.61}$$

where $\overline{\eta}_{\text{17QD18}} = (E_{\text{FQD}} - \overline{E}_{\text{18QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\overline{E}_{\text{QD33}}(1 + \alpha \overline{E}_{\text{QD33}}) = [\overline{\omega}_1(t_x)^2 + \overline{\omega}_2(t_y)^2 + \overline{\omega}_3(t_z)^2 + 2\overline{\omega}_4(t_y t_z)] \tag{5.62}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{\text{47QD19}}) \tag{5.63}$$

where, $\eta_{47QD19} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{QD33})$.

5.2.10 The Germanium

(a) **Model of Cardona et al.**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{P}_x = L_{70}(\overline{E}_{\text{20QBHD}}, n_y, n_z, \eta_g) \tag{5.64}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{\text{18QD20}}) \tag{5.65}$$

where $\overline{\eta}_{\text{18QD20}} = (E_{\text{FQD}} - \overline{E}_{\text{20QBHD}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\overline{E}_{\mathrm{QD30}} = -\frac{E_{g_0}}{2} + \frac{\hbar^2}{2m^*_{||}}(\beta_z)^2\left[\frac{E^2_{g_0}}{4} + E_{g_0}\left(\frac{\hbar^2}{2m^*_{||}}\right)[\{(t_x)^2 + (t_y)^2\}]\right]^{1/2} \tag{5.66}$$

$$n_{0D} = \overline{G}_8 F_{-1}(\eta_{48\mathrm{QD21}}) \tag{5.67}$$

where, $\eta_{48\mathrm{QD21}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\mathrm{QD30}})$.

(b) **Model of Wang and Ressler**

The $0D$ HDDR and n_0 assume the form [30]

$$\overline{P}_x = \beta_{70}(\overline{E}_{24\mathrm{QBHD24}}, n_y, n_z, \eta_g) \tag{5.68}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{18\mathrm{QD22}}) \tag{5.69}$$

where $\overline{\eta}_{18\mathrm{QD22}} = (E_{\mathrm{FQD}} - \overline{E}_{24\mathrm{QBHD24}})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\begin{aligned}\overline{E}_{\mathrm{QD40}} = {} & \frac{\hbar^2}{2m^*_{||}}(t_z)^2 + \frac{\hbar^2}{2m^*_{\perp}}\{(t_x)^2 + (t_y)^2\} - \overline{c}_1\left(\frac{\hbar^2}{2m^*_{\perp}}\right)\{(t_x)^2 + (t_y)^2\} \\ & - \overline{d}_1\left(\frac{\hbar^2}{2m^*_{\perp}}\right)\{(t_x)^2 + (t_y)^2\}\left(\frac{\hbar^2}{2m^*_{||}}\right)(\beta_z)^2 - \overline{e}_1\left(\frac{\hbar^2}{2m^*_{||}}\right)(t_z)^4\end{aligned} \tag{5.70}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{49\mathrm{QD23}}) \tag{5.71}$$

where, $\eta_{49\mathrm{QD23}} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\mathrm{QD40}})$.

5.2.11 The Gallium Antimonide

The DRs of n-GaSb have been studied by Seiler et al. [27], Mathuret al. [28] and Zhang [29] and he studies the ES accordingly.

(i) Using the Seiler model, the $0D$ HDDR and n_0 assume the form [30]

$$\begin{aligned}\gamma_3(\bar{E}_{30\mathrm{QBHD11}}, \eta_g) = {} & \left[-\frac{E_{g_0}}{2} + \frac{E_{g_0}}{2}[1 + \alpha_4 k_1^2]^{1/2}\right. \\ & \left. + \frac{\overline{\zeta}_0 \hbar^2 k_1^2}{2m_0} + \frac{\overline{v}_0 \theta_1(k_1)\hbar^2}{2m_0} \pm \frac{\overline{w}_0 \theta_2(k_1)\hbar^2}{2m_0}\right]\end{aligned} \tag{5.72}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{19QD24}) \tag{5.73}$$

where $\overline{\eta}_{19\text{QD}24} = (E_{\text{FQD}} - \overline{E}_{30\text{QBHD}11})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\overline{E}_{\text{sQD}} = \left[-\frac{E_{g_0}}{2} + \frac{E_{g_0}}{2}[1 + \alpha_4 k_1^2]^{1/2} + \frac{\overline{\zeta_0}\hbar^2 k_1^2}{2m_0} + \frac{\overline{v_0}\theta_1(k_1)\hbar^2}{2m_0} \pm \frac{\overline{w_0}\theta_2(k_1)\hbar^2}{2m_0} \right] \tag{5.74}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{50\text{QD}25}) \tag{5.75}$$

where, $\eta_{50\text{QD}25} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{sQD}})$.

(ii) Using Mathur model, the $0D$ HDDR and n_0 assume the form [30]

$$\overline{G}_7 = I_{36}(\overline{E}_{30\text{QBHDM}}, \eta_g) \tag{5.76}$$

$$n_0 = \overline{G}_8 F_{-1}(\overline{\eta}_{20\text{QD}26}) \tag{5.77}$$

where $\overline{\eta}_{20\text{QD}26} = (E_{\text{FQD}} - \overline{E}_{30\text{QBHDM}})\,(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$I_{36}(\overline{E}_{\text{QD}60}) = \overline{G}_7 \tag{5.78}$$

$$n_0 = \overline{G}_8 F_{-1}(\eta_{51\text{QD}27}) \tag{5.79}$$

where, $\eta_{51\text{QD}27} \equiv (k_B T)^{-1}(E_{F0D} - \overline{E}_{\text{QD}60})$.

(iii) Using Zhang model, The $0D$ HD DR a and n_0 assume the form [30]

$$\begin{aligned}\gamma_3(\overline{E}_{30\text{QBHD}22}, \eta_g) = {} & [\overline{E_1} + \overline{E_2}Z_1(k_1)]k_1^2 + [\overline{E_3} + \overline{E_4}Z_1(k_1)]k_1^4 \\ & + k_1^6[\overline{E_5} + \overline{E_6}Z_1(k_1) + \overline{E_7}Z_2(k_1)]\end{aligned} \tag{5.80}$$

$$n_0 = \frac{\overline{G}_8 F_{-1}(\overline{\eta}_{21\text{QD}28})}{2} \tag{5.81}$$

where $\overline{\eta}_{21\text{QD}28} = (E_{\text{FQD}} - \overline{E}_{30\text{QBHD}36})(k_B T)^{-1}$.

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\begin{aligned}\bar{E}_{100\text{QD}} = {} & [\overline{E_1} + \overline{E_2}Z_1(k_1)]k_1^2 + [\overline{E_3} + \overline{E_4}Z_1(k_1)]k_1^4 \\ & + k_1^6[\overline{E_5} + \overline{E_6}Z_1(k_1) + \overline{E_7}Z_2(k_1)]\end{aligned} \tag{5.82}$$

$$n_0 = \frac{\overline{G}_8}{2} F_{-1}(\eta_{101QD28}) \tag{5.82'}$$

where, $\eta_{101QD28} = (k_B T)^{-1}(E_{F0D} - \overline{E}_{100QD})$

5.2.12 The II-V Materials

The $0D$ HDDR and n_0 assume the form [30]

$$\begin{aligned}\gamma_3(\bar{E}_{150\text{QBHD}_{,\pm}}, \eta_g) = {} & A_{10}(t_x)^2 + A_{11}(t_y)^2 + A_{12}(t_z)^2 + A_{13}(t_x)^2 \\ & \pm [A_{14}(t_x)^2 + A_{15}(t_y)^2 + A_{16}(t_z)^2 \\ & + A_{17}(t_x)^2 + A_{18}(t_y)^2 + A_{19}]\end{aligned} \tag{5.83}$$

$$n_0 = \frac{\overline{G}_8}{2} F_{-1}(\eta_{51,31,\pm}) \tag{5.83'}$$

where, $\eta_{51,31,\pm} = (k_B T)^{-1}(E_{F0D} - \overline{E}_{150QBHD\pm})$

For $\eta_g \to 0$, the $0D$ DR and n_0 get simplified as

$$\begin{aligned}\overline{E}_{\text{QD80}_{,\pm}} = {} & A_{10}(t_x)^2 + A_{11}(t_y)^2 + A_{12}(t_z)^2 + A_{13}(t_x) \\ & \pm [(A_{14}(t_x)^2 + A_{15}(t_y)^2 + A_{16}(t_z)^2 + A_{17}(t_x)^2 + A_{18}(t_y)^2 + A_{19}^2]^{1/2}\end{aligned} \tag{5.84}$$

$$n_0 = \frac{\overline{G}_8 F_{-1}(\eta_{54\text{QD31}})}{2} \tag{5.85}$$

where, $\eta_{54QD31} = (k_B T)^{-1}(E_{F0D} - \overline{E}_{QD80_{,\pm}})$.

5.3 Summary and Conclusion

- **In QDs, all the components of the wave vector are being quantized leading to discreet systems of energies. In QDs, there remain no free carrier states in between any two allowed sets of size-quantized levels and band structure controls all the physical properties leading to radical new directions.**

We have not given any plot of n_0 for any HD QDs as considered here with respect to any physical variable since we believe our able readers will plot the same and enjoy intricate physics, computational analysis and advanced mathematics and will create new problems both theoretical and experimental.

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Chapter 6
The HUP and the ES in Doping Superlattices of HD Non-parabolic Semiconductors

6.1 Introduction

The doping superlattices in recent years find various important technological applications in low-dimensional electronics [1–20]. The present chapter discusses the ES chronologically for all the cases as given in the content of this chapter in Sect. 6.2 and the result and discussions in Sect. 6.3, respectively.

6.2 Mathematical Basis

6.2.1 The Nonlinear Optical Semiconductors

The 2*D* HDDR is given by [21]

$$k_s^2 = G_{21\mathrm{HD}}(E, \eta_g, n_i) \tag{6.1}$$

The EEM and n_{2D} become

$$m^*(\overline{E}_{\mathrm{FnHD}}, n_i, \eta_g) = R.P.\left(\frac{\hbar^2}{2}\right) G'_{21\mathrm{HD}}(\overline{E}_{\mathrm{FnHD}}, n_i, \eta_g) \tag{6.2}$$

$$n_{2D} = R.P.\,\overline{n}_{2D} \tag{6.3}$$

where

$$\overline{n}_{2D} = \xi_8[G_{21\mathrm{HD}}(E_{\mathrm{FnHD}}, \eta_g, n_i)] \tag{6.4}$$

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in which $\xi_8 = \left[\left(\frac{g_v}{2\pi}\right)\sum_{n_i=0}^{n_{i\max}}\right.$

When $\eta_g \rightarrow 0$ the 2D DR can be written as [21]

$$k_s^2 = T_{81}(E, n_i) \tag{6.5}$$

The EEM and n_{2D} become

$$m^*(E_{\mathrm{Fn}}, n_i) = \left(\frac{\hbar^2}{2}\right)[[T_{81}(E, n_i)]|_{E=\overline{E}_{\mathrm{Fn}}}]' \tag{6.6}$$

$$n_{2D} = \xi_8[T_{81}(\overline{E}_{\mathrm{Fn}}, n_i)]. \tag{6.7}$$

6.2.2 The III-V and Opto-Electronic Semiconductor

(a) The 2*D* HDDR for three-band Kane model assumes the form [21]

$$k_s^2 = G_{23\mathrm{HD}}(E, \eta_g, n_i) \tag{6.8}$$

The EEM and n_{2D} become

$$m^*(E_{\mathrm{FnHD}}, n_i\eta_g) = \mathrm{R.P.}\left(\frac{\hbar^2}{2}\right)G'_{23\mathrm{HD}}(E_{\mathrm{FnHD}}, \eta_g, n_i) \tag{6.9}$$

$$n_{2D} = R.P.\ \overline{n}_{2D} \tag{6.10}$$

where

$$\overline{n}_0 = \xi_8[G_{23\mathrm{HD}}(E_{\mathrm{FnHD}}, n_i, \eta_g)] \tag{6.11}$$

When $\eta_g \rightarrow 0$ the 2D DR takes the form [21]

$$\frac{\hbar^2 k_s^2}{2m_c} = T_{83}(E, n_i) \tag{6.12}$$

where $T_{83}(E, n_i) \equiv \left[I_{11}(E) - \left(n_i + \frac{1}{2}\right)\hbar\omega_{19}\right]$ and $\omega_{19}(E) \equiv \left[\frac{n_0|e|^2}{\varepsilon_{sc}I'_{11}(E)m_c}\right]^{1/2}$

The EEM and n_{2D} become

$$m^*(E, n_i) = m_c[[T_{83}(E, n_i)]\big|_{E=E_{\mathrm{Fn}}}]' \tag{6.13}$$

$$n_{2D} = \frac{2m_c}{\hbar^2}\xi_8[T_{83}(E_{\text{FnHD}}, n_i)] \tag{6.14}$$

(b) **The Two-Band Model of Kane**

The 2*D* HDDR is given by [21]

$$k_s^2 = G_{25\text{HD}}(E, \eta_g, n_i) \tag{6.15}$$

where, $G_{25\text{HD}}(E, \eta_g, n_i) = \frac{2m_c}{\hbar^2}[\gamma_2(E, \eta_g) - (n_i + \frac{1}{2})\hbar\omega_{10\text{HD}}(E, \eta_g)]$ and $\omega_{10\text{HD}}(E) \equiv (\frac{n_0|e|^2}{\varepsilon_{sc}\gamma_2'(E,\eta_g)m_c})^{1/2}$

The EEM and n_{2D} become

$$m^*(E_{\text{FnHD}}, n_i, \eta_g) = \left(\frac{\hbar^2}{2}\right)G'_{25\text{HD}}(E_{\text{FnHD}}, \eta_g, n_i) \tag{6.16}$$

$$n_{2D} = \frac{2m_c}{\hbar^2}\xi_8[G_{25\text{HD}}(E_{\text{FnHD}}, n_i, \eta_g)] \tag{6.17}$$

When $\eta_g \to 0$ the (6.15) gets simplified as

$$E(1 + \alpha E) = \left(n_i + \frac{1}{2}\right)\hbar\omega_{20}(E) + \frac{\hbar^2 k_s^2}{2m_c} \tag{6.18}$$

The EEM and n_{2D} become [21]

$$m^*(E_{Fn}, n_i) = m_c R_{182}(E, n_i)\big|_{E=E_{\text{Fn}}} \tag{6.19}$$

$$n_{2D} = \frac{2m_c}{\hbar^2}\xi_8[T_{831}(E_{\text{FnHD}}, n_i)] \tag{6.20}$$

where, $T_{831}(\overline{E}_{\text{Fn}}, n_i) \equiv \left[(1 + \alpha\overline{E}_{\text{Fn}})(\overline{E}_{\text{Fn}}) - \left(n_i + \frac{1}{2}\right)\hbar\omega_9(\overline{E}_{\text{Fn}})\right]$

(c) **Parabolic Energy Bands**

The 2*D* HDDR becomes [21]

$$k_s^2 = G_{27\text{HD}}(E, \eta_g, n_i) \tag{6.21}$$

where, $G_{27\text{HD}}(E, \eta_g, n_i) = \frac{2m_c}{\hbar^2}\left[\gamma_3(E, \eta_g) - \left(n_i + \frac{1}{2}\right)\hbar\omega_{11\text{HD}}(E, \eta_g)\right]$

The EEM and n_{2D} become

$$m^*(E_{\text{FnHD}}, n_i, \eta_g) = \left(\frac{\hbar^2}{2}\right)G'_{27\text{HD}}(E_{\text{FnHD}}, n_i, \eta_g) \tag{6.22}$$

$$n_{2D} = \xi_8[G_{27\mathrm{HD}}(E_{\mathrm{FnHD}}, n_i, \eta_g)] \tag{6.23}$$

When $\eta_g \to 0$ the (6.21) gets simplified as

$$E = \left(n_i + \frac{1}{2}\right)\hbar\omega_{21} + \frac{\hbar^2 k_s^2}{2m_c} \tag{6.24}$$

where, $\omega_{21}(E) \equiv \left(\frac{n_0|e|^2}{d_0\varepsilon_{\mathrm{sc}}m_c}\right)^{1/2}$

The EEM and n_0 become

$$m^*(E_{\mathrm{Fn}}, n_i) = m_c \tag{6.25}$$

$$n_{2D} = \frac{2m_c}{\hbar^2}\xi_8(E_{\mathrm{Fn}} - E_{4ni}). \tag{6.26}$$

6.2.3 The II-VI Semiconductors

The 2*D* HDDR becomes [21]

$$\gamma_3(E, \eta_g) = a_0' k_s^2 + \left(n_i + \frac{1}{2}\right)\hbar\omega_{30}(E, \eta_g) \pm \overline{\lambda}_0 k_s, \tag{6.27}$$

The EEM and n_{2D} become

$$m^*(E_{\mathrm{FnHD}}, n_i, \eta_g) = m_\perp^*\Big\{1 - \overline{\lambda}_0\Big[(\overline{\lambda}_0)^2 + 4a_0'\gamma_3(E_{\mathrm{FnHD}}, \eta_g) - 4a_0'\left(n_i + \frac{1}{2}\right)\hbar^2\omega_{30}(E_{\mathrm{FnHD}}, \eta_g)\Big]^{-1/2}\Big\}\gamma_3'(E_{\mathrm{FnHD}}, \eta_g) \tag{6.28}$$

$$n_{2D} = \frac{1}{2(a_0')^2}\xi_8[G_{30\mathrm{HD}}(E_{\mathrm{FnHD}}, n_i, \eta_g)] \tag{6.29}$$

When $\eta_g \to 0$ the (6.27) gets simplified as

$$E = a_0' k_s^2 + \left(n_i + \frac{1}{2}\right)\hbar\overline{\omega}_{10} \pm \overline{\lambda}_0 k_s, \overline{\omega}_{10} = \left(\frac{n_0|e|^2}{d_0\varepsilon_{\mathrm{sc}}m_{||}^*}\right)^{1/2} \tag{6.30}$$

The EEM and n_{2D} become

$$m^*(E_{\mathrm{Fn}}, n_i) = m_\perp^* \left\{ 1 - \overline{\lambda}_0 \left[(\overline{\lambda}_0)^2 + 4a_0' E_{\mathrm{Fn}} - 4a_0' \left(n_i + \frac{1}{2} \right) \hbar^2 \overline{\omega}_{10} \right]^{-1/2} \right\} \tag{6.31}$$

$$n_{2D} = \frac{2m_c}{\hbar^2} \xi_8 [E_{\mathrm{Fn}} - E_{8ni}]. \tag{6.32}$$

6.2.4 The IV-VI Semiconductors

The 2*D* HDDR becomes [21]

$$k_s^2 = \delta_{15}(E, \eta_g, n_i) \tag{6.33}$$

The EEM and n_{2D} become

$$m^*(E_{FnHD}, n_i, \eta_g) = \left(\frac{\hbar^2}{2} \right) \delta_{15}'(E_{FnHD}, \eta_g, n_i) \tag{6.34}$$

$$n_{2D} = \frac{2m_c}{\hbar^2} \xi_8 [G_{32\mathrm{HD}}(E_{\mathrm{FnHD}}, \eta_g, n_i)]. \tag{6.35}$$

6.2.5 The Stressed Kane-Type Semiconductors

The 2*D* HDDR becomes [21]

$$P_{11}(E, \eta_g) k_x^2 + Q_{11}(E, \eta_g) k_y^2 + S_{11}(E, \eta_g) \delta_{19}(E, \eta_g, n_i) = 1 \tag{6.36}$$

The EEM gets simplified as

$$m^*(E_{FnHD}, n_i, \eta_g) = \left(\frac{\hbar^2}{2} \right) \delta_{20}'(E_{FnHD}, \eta_g, n_i) \tag{6.37}$$

where, $\delta_{20}(E_{\mathrm{FnHD}}, \eta_g, n_i) = \frac{[1 - S_{11}(E_{\mathrm{FnHD}}, \eta_g, n_i) \delta_{19}(E_{\mathrm{FnHD}}, \eta_g, n_i)]}{\sqrt{P_{11}(E_{\mathrm{FnHD}}, \eta_g, n_i) Q_{11}(E_{\mathrm{FnHD}}, \eta_g)}}$

The n_{2D} can be written as

$$n_{2D} = \xi_8 [G_{34\mathrm{HD}}(E_{\mathrm{FnHD}}, \eta_g, n_i)] \tag{6.38}$$

When $\eta_g \rightarrow 0$ the 2*D* HDDR can be written as [21]

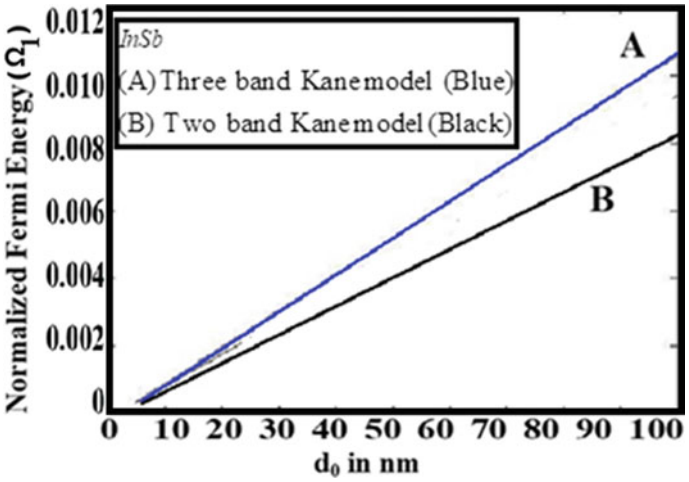

Fig. 6.1 Ω_1 versus superlattice period (d_0) for InSb HD DSL where $n_o = 10^{23}$ m^{-3} (blue—3 band, black—2 Band)

$$\frac{k_x^2}{[\overline{a}_0(E)]^2} + \frac{k_y^2}{[\overline{b}_0(E)]^2} + \frac{1}{[\overline{c}_0(E)]^2}\frac{2m_z^*(0)}{\hbar}(n_i + \frac{1}{2})\omega_{12}(E) = 1 \tag{6.39}$$

The EEM and n_{2D} become

$$m^*(E_{\mathrm{Fn}}, n_i) = \left(\frac{\hbar^2}{2}\right) R_{85}(E, n_i)\Big|_{E=E_{\mathrm{Fn}}} \tag{6.40}$$

$$n_{2D} = \xi_8[C_3(E_{\mathrm{FnHD}}, n_i)]. \tag{6.41}$$

6.3 Result and Discussions

The use of appropriate equations leads us to generate the plots of Ω_1 in Figs. 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9 and 6.10 with respect to various variables, respectively. From the figures, the following features follow:

1. From Figs. 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7 and 6.8, we note that, the Ω_1 increases with increasing d_0 and n_0 of HDDSL as considered here with different numerical magnitudes.
2. Figs. 6.9 and 6.10 illustrated that Ω_1 decreases with decreasing x for both types of ternary and quaternary HDDSL.

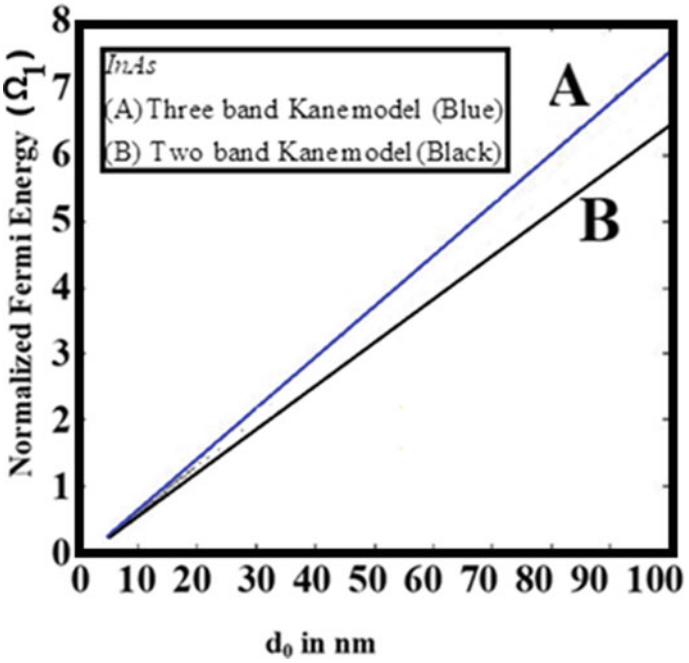
InAs
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
A
B
Normalized Fermi Energy (Ω1)
0 1 2 3 4 5 6 7 8
0 10 20 30 40 50 60 70 80 90 100
d0 in nm

Fig. 6.2 All plots of Fig. 6.1 for InAS HD DSL

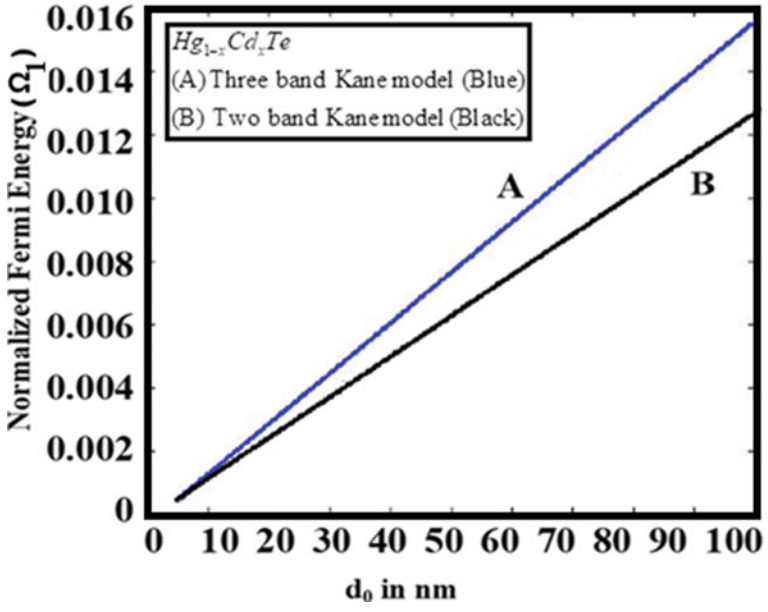
Hg1-xCdxTe
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
A
B
Normalized Fermi Energy (Ω1)
0 0.002 0.004 0.006 0.008 0.010 0.012 0.014 0.016
0 10 20 30 40 50 60 70 80 90 100
d0 in nm

Fig. 6.3 All plots of Fig. 6.1 for $Hg_{1-x}Cd_xTe$ HD DSL

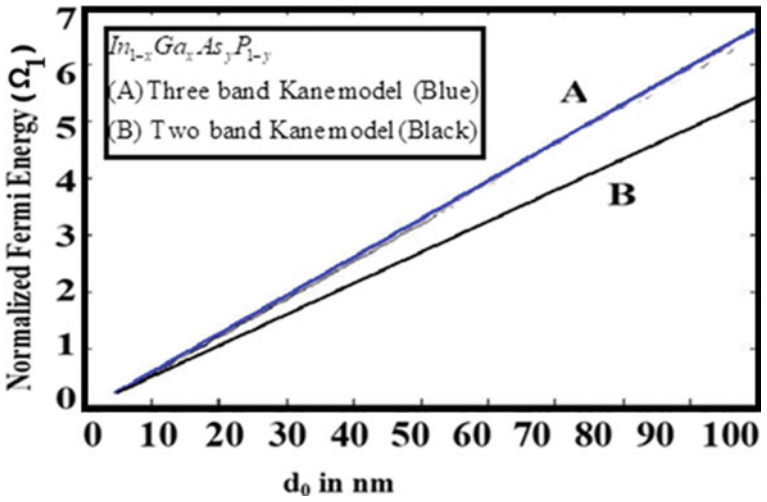
$In_{1-x}Ga_xAs_yP_{1-y}$
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
A
B
Normalized Fermi Energy (Ω_1)
0 1 2 3 4 5 6 7
0 10 20 30 40 50 60 70 80 90 100
d_0 in nm

Fig. 6.4 All plots of Fig. 6.1 for $In_{1-x}Ga_xAs_yP_{1-y}$ HD DSL

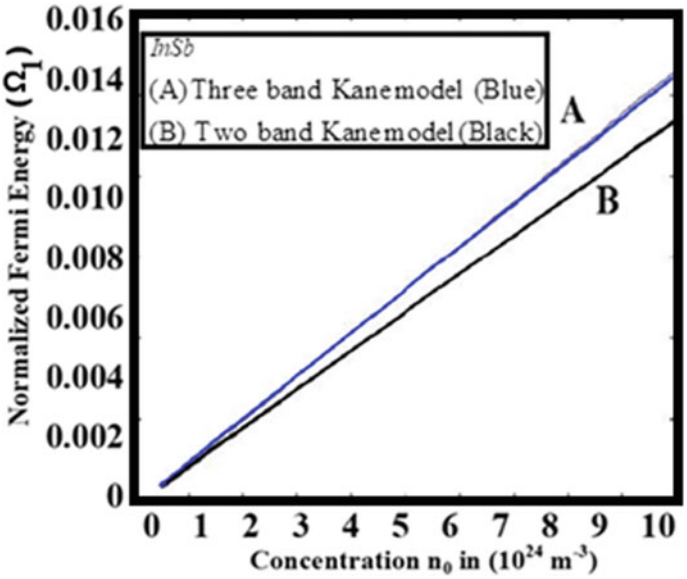
InSb
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
A
B
Normalized Fermi Energy (Ω_1)
0 0.002 0.004 0.006 0.008 0.010 0.012 0.014 0.016
0 1 2 3 4 5 6 7 8 9 10
Concentration n_0 in (10^{24} m^{-3})

Fig. 6.5 Plot of the Ω_1 versus n_0 for InSb HD DSL for three (**a**) and two (**b**) band Kane model, where $d_o = 1$ nm

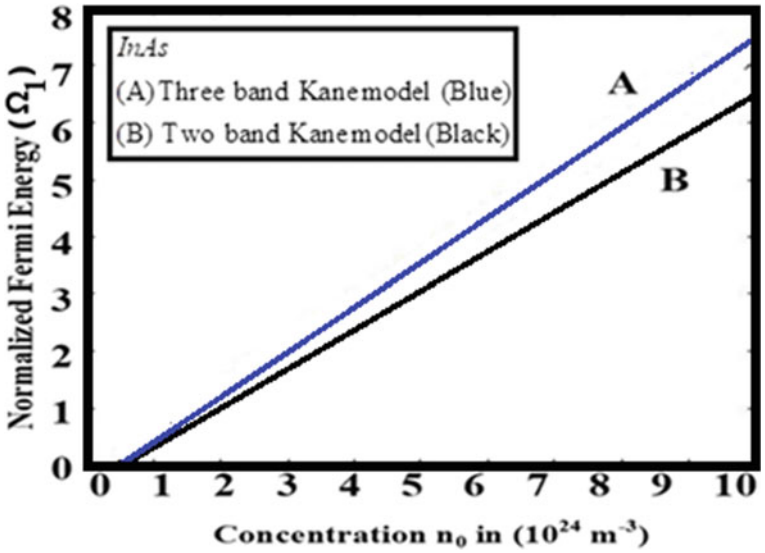

Fig. 6.6 All plots of Fig. 6.5 for InAS HD DSL

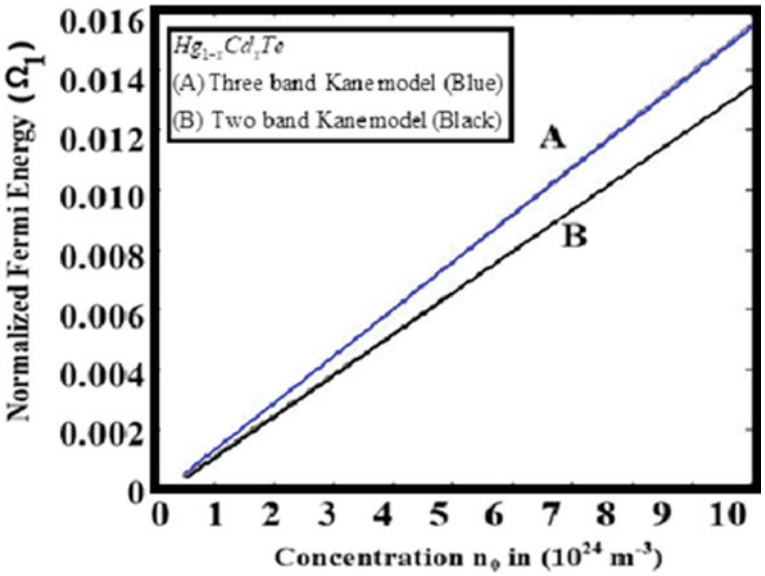

Fig. 6.7 All plots of Fig. 6.5 for $Hg_{1-x}Cd_xTe$ HD DSL

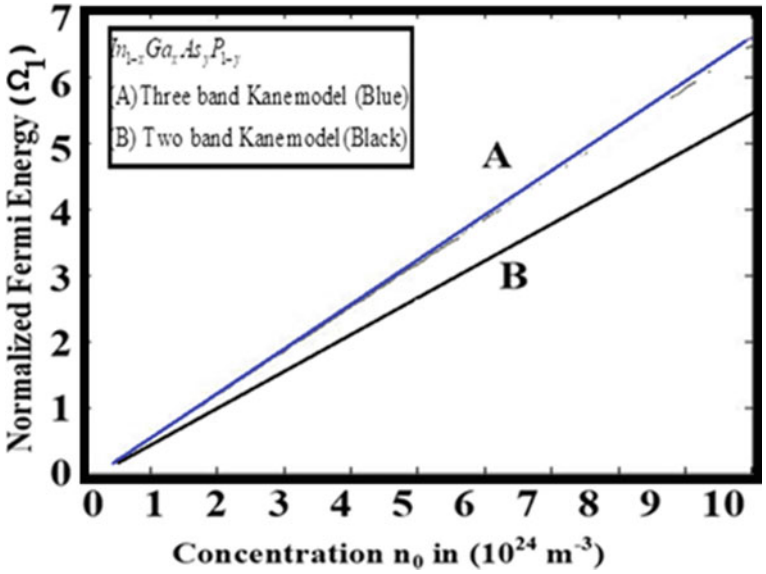
$In_{1-x}Ga_xAs_yP_{1-y}$
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
A
B
Normalized Fermi Energy (Ω_1)
Concentration n_0 in (10^{24} m^{-3})
0 1 2 3 4 5 6 7
0 1 2 3 4 5 6 7 8 9 10

Fig. 6.8 All plots of Fig. 6.5 for $In_{1-x}Ga_xAs_yP_{1-y}$ HD DSL

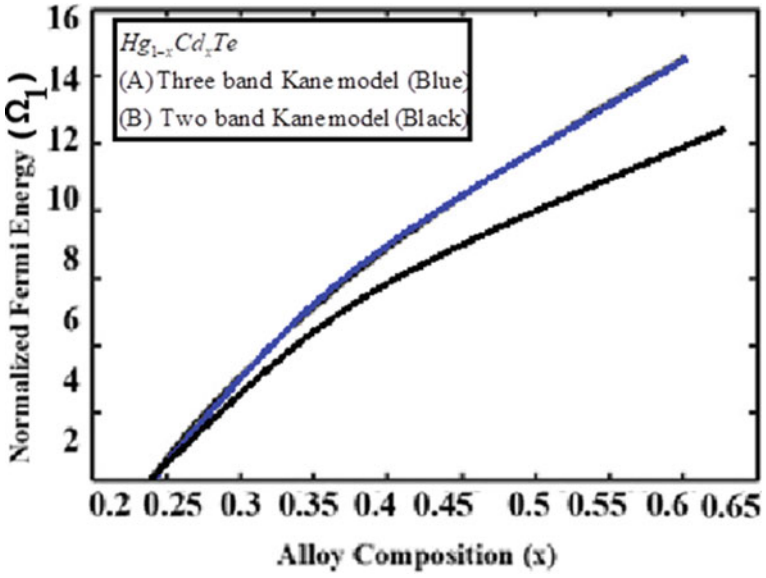
$Hg_{1-x}Cd_xTe$
(A) Three band Kane model (Blue)
(B) Two band Kane model (Black)
Normalized Fermi Energy (Ω_1)
Alloy Composition (x)
2 4 6 8 10 12 14 16
0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65

Fig. 6.9 Plot of the Ω_1 versus x for $Hg_{1-x}Cd_xTe$ HD DSL where $d_o = 1$ nm and $n_o = 10^{23}$ m^{-3}

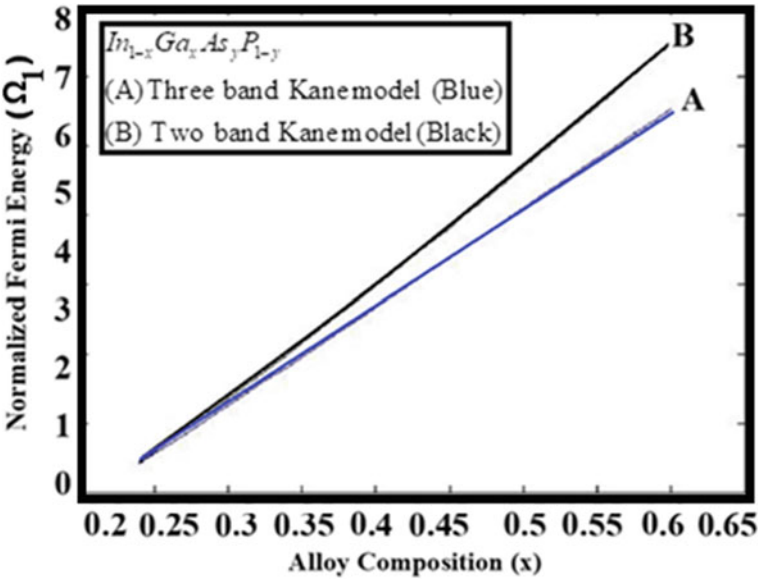

Fig. 6.10 All plots of Fig. 6.5 for $In_{1-x}Ga_xAs_yP_{1-y}$ HD DSL

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Chapter 7
The HUP and the ES in Accumulation of Non-parabolic Semiconductors

7.1 Introduction

The quantum MOSFETs are made of different technologically important materials, and the 2D carrier transport in such structures has been studied widely [1–20], although the same for accumulation layers (ALs) have relatively been less investigated [21]. Here, we study the ES in ALs chronologically in accordance with the content of this chapter. In Sect. 7.3, we study a single ES dependent electronic property namely the DMR in this context.

7.2 Mathematical Basis

7.2.1 The NonLinear Optical Semiconductors

The 2D HDDR for ALs for $(E/E_g) \ll 1$ gets simplified as [21]

$$\frac{\hbar^2 k_s^2}{2m_{||}^*} = L_6(E, i, \eta_g) \tag{7.1}$$

The EEM can be written as

$$m^*(E_f', i, \eta_g) = m_{||}^* \, R.\ P.[L_6'(E_f', i, \eta_g)] \tag{7.2}$$

where,
$E_f' = eV_g - \frac{e^2 n_s d_{ex}}{\varepsilon_{ox}} + E_{FB}$,
The E_{FB} is connected with n_B through the following equation

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$$n_B = \frac{2g_v}{(2\pi)^3}\frac{2m_{\perp}^*\sqrt{2m_{||}^*}}{\hbar^3}\, R.\ P.\left[T_{22}\left(E_{FB},\eta_g\right)\sqrt{T_{21}\left(E_{FB},\eta_g\right)}\right] \tag{7.3}$$

The n_s is given by

$$n_s = R.P.\,\bar{n}_s \tag{7.4}$$

where,

$$\bar{n}_s = 2g_v\sum_{i=0}^{i_{\max}}\left[\left[\frac{m_{\perp}^*}{2\pi\hbar^2}L_6\left(E_f',i,\eta_g\right)\right] + \frac{1}{(2\pi)^3}\frac{2m_{\perp}^*\sqrt{2m_{||}^*}}{\hbar^3}t_i\left[T_{22}\left(E_{FB},\eta_g\right)\sqrt{T_{21}\left(E_{FB},\eta_g\right)}\right]\right]$$

$t_i = \frac{E_{i\,\max}}{eF_s(1+i_{\max})}$, $E_{i\,\max}$, is determined through the Real part of the following:

$$T_{21}(E_{i\,\max},\eta_g) - L_3(E_{i\,\max},i_{\max},\eta_g) = 0 \tag{7.5}$$

7.2.2 The III-V and Opto-Electronic Materials

(a) The 2D HDDR for ALs for $(E/E_g) \ll 1$ gets simplified as

$$T_{90}(E,\eta_g) = \frac{\hbar^2k_s^2}{2m_c} + S_i\left[\frac{\hbar|e|F_s[T_{90}(E,\eta_g)]'}{\sqrt{2m_c}}\right]^{2/3} \tag{7.6}$$

The EEM and n_s can, become

$$m^*(E_f',i,\eta_g) = m_c\, R.P.\, P_{3HD}'(E_f',i,\eta_g) \tag{7.7}$$

$$n_s = R.\,P.\,\bar{n}_s \tag{7.8}$$

where,

$$\bar{n}_s = g_v\sum_{i=0}^{i_{\max}}\left[\left[\frac{m_c}{\pi\hbar^2}P_{3HD}\left(E_f',i,\eta_g\right)\right]\right.$$

$$+\frac{1}{3\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} t_i\left[T_{90}\left(E_{FB},\eta_g\right)\right]^{3/2}\Bigg]$$

and

$$n_B = \frac{g_v}{3\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} R.P.\left[T_{90}\left(E_{FB},\eta_g\right)\right]^{3/2} \tag{7.9}$$

(b) The 2D HDDR for accumulation layers for $(E/E_g) \ll 1$ assumes the form

$$\gamma_2(E,\eta_g) = \frac{\hbar^2 k_s^2}{2m_c} + S_i\left[\frac{\hbar|e|F_s\left[\gamma_2(E,\eta_g)\right]'}{\sqrt{2m_c}}\right]^{2/3} \tag{7.10}$$

The EEM and n_S become

$$m^*\left(E_f',i,\eta_g\right) = m_c P_{3HD1}'\left(E_f',i,\eta_g\right) \tag{7.11}$$

$$n_S = g_v \sum_{i=0}^{i_{\max}}\Bigg[\left[\frac{m_c}{\pi\hbar^2}P_{3HD1}\left(E_f',i,\eta_g\right)\right] + \frac{1}{3\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} t_i\left[\gamma_2\left(E_{FB},\eta_g\right)\right]^{3/2}\Bigg] \tag{7.12}$$

and

$$n_B = \frac{g_v}{3\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\left[\gamma_2\left(E_{FB},\eta_g\right)\right]^{3/2} \tag{7.13}$$

(c) For $\alpha \to 0$, the 2D DR becomes

$$\gamma_3(E,\eta_g) = \frac{\hbar^2 k_s^2}{2m_c} + S_i\left[\frac{\hbar|e|F_s[\gamma_3(E,\eta_g)]'}{\sqrt{2m_c}}\right]^{2/3} \tag{7.14}$$

The EEM becomes

$$m^*\left(E_f',i,\eta_g\right) = m_c P_{3HD2}'\left(E_f',i,\eta_g\right) \tag{7.15}$$

where,

$$P_{3HD2}\left(E_f',i,\eta_g\right) = \left[\gamma_3\left(E_f',\eta_g\right) - S_i\left[\frac{\hbar|e|F_s\left[\gamma_3\left(E_f',\eta_g\right)\right]'}{\sqrt{2m_c}}\right]^{2/3}\right]$$

The n_s is given by

$$n_s = g_v \sum_{i=0}^{i_{\max}} \left[\left[\frac{m_c}{\pi \hbar^2} P_{3HD2}\left(E'_f, i, \eta_g\right) \right] + \frac{1}{3\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} t_i \left[\gamma_3\left(E_{FB}, \eta_g\right) \right]^{3/2} \right] \tag{7.16}$$

in which

$$n_B = \frac{g_v}{3\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \left[\gamma_3\left(E_{FB}, \eta_g\right) \right]^{3/2} \tag{7.17}$$

7.2.3 The II-VI Semiconductors

The EEM becomes

$$m^*(E'_f, \eta_g, i) = m^*_{\perp} \Delta_{10}(E'_f, \eta_g, i) \tag{7.18}$$

The n_s becomes

$$n_s = g_v \sum_{i=0}^{i_{\max}} \left[\left[\left(\frac{m^*_{\perp}}{\pi \hbar^2} \right) \left[\Delta_{10}\left(E'_f, i, \eta_g\right) \right] \right] + \frac{t_i}{2} \left(\frac{1}{\pi b'_0} \right)^{3/2} \left(\frac{b'_0}{a'_0} \right) \left[\frac{4}{3\sqrt{\pi}} (E_{FB})^{3/2} + \frac{\left(\bar{\lambda}_0\right)^2}{2a'_0} \left(\frac{2}{\sqrt{\pi}} \right) (E_{FB})^{1/2} \right] \right] \tag{7.19}$$

in which

$$n_B = \frac{g_v}{2} \left(\frac{1}{\pi b'_0} \right)^{3/2} \left(\frac{b'_0}{a'_0} \right) \left[\frac{4}{3\sqrt{\pi}} (E_{FB})^{3/2} + \frac{\left(\bar{\lambda}_0\right)^2}{2a'_0} \frac{2}{\sqrt{\pi}} (E_{FB})^{1/2} \right] \tag{7.20}$$

7.2.4 The IV-VI Semiconductors

The EEM becomes

$$m^*\left(E, i, \eta_g\right) = \frac{\hbar^2}{2} \theta'_4\left(E'_f, i, \eta_g\right) \tag{7.21}$$

The n_s becomes

$$n_s = R.\ P.\ \overline{n}_s \tag{7.22}$$

where,

$$\overline{n}_S = g_v \sum_{i=0}^{i_{\max}} \left[\theta_4\left(E'_f, i, \eta_g\right) + \frac{t_i}{3\pi^2} \left[F_1\left(E_{FB}, \eta_g\right) \sqrt{F_2\left(E_{FB}, \eta_g\right)} \right]^{-1} \right] \tag{7.23}$$

in which

$$n_B = g_v\, R.\ P. \left[\frac{1}{3\pi^2} \left[F_1\left(E_{FB}, \eta_g\right) \sqrt{F_2\left(E_{FB}, \eta_g\right)} \right]^{-1} \right] \tag{7.24}$$

7.2.5 The Stressed Kane-Type Materials

The EEM becomes

$$m^*\left(E'_f, i, \eta_g\right) = \frac{\hbar^2}{2} \theta'_{43}\left(E'_f, i, \eta_g\right) \tag{7.25}$$

The n_s becomes

$$n_s = \overline{n}_s \tag{7.26}$$

where,

$$\bar{n}_S = g_v \sum_{i=0}^{i_{\max}} \left[\theta_{43}\left(E'_f, i, \eta_g\right) + \frac{t_i}{3\pi^2} \left[f_1\left(E_{FB}, \eta_g\right) f_2\left(E_{FB}, \eta_g\right) f_3\left(E_{FB}, \eta_g\right) \right]^{-1/2} \right]$$

and

$$n_B = g_v \left[f_1\left(E_{FB}, \eta_g\right) f_2\left(E_{FB}, \eta_g\right) f_3\left(E_{FB}, \eta_g\right) \right]^{-1/2} \tag{7.27}$$

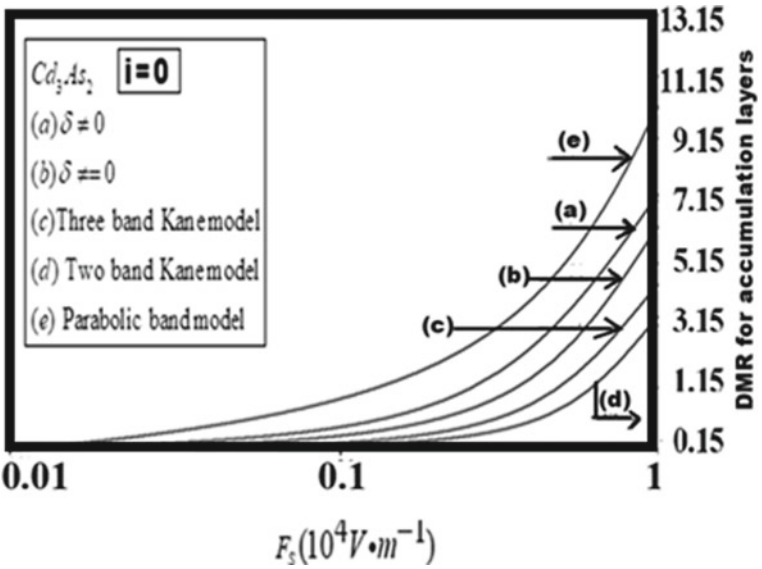

Fig. 7.1 The DMR in accumulation layers against F_s for five cases of the indicated material

7.3 Results and Discussion

The DMR has been plotted in this case with different physical variables in Figs. 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7 and 7.8 to illustrate at least a single electronic property dependent on ES in this context. From the said figures, we note the following points:

1. From Figs. 7.1 and 7.2, we observe that the DMR in accumulation layers of Cd_3As_2 and $CdGeAs_2$ increase with increasing F_S and the influence of δ is to enhance the DMR for all the cases.
2. From Figs. 7.3, 7.4, 7.5 and 7.6, we note that the DMR is monotonic increasing functions of F_S for inversion or accumulation layers of InAs, InSb, $Hg_{1-x}Cd_xTe$ and $In_{1-x}Ga_xAs_yP_{1-y}$, respectively.
3. From Figs. 7.7 and 7.8, we note that the DMR is a monotonic decreasing function of x with different slopes.

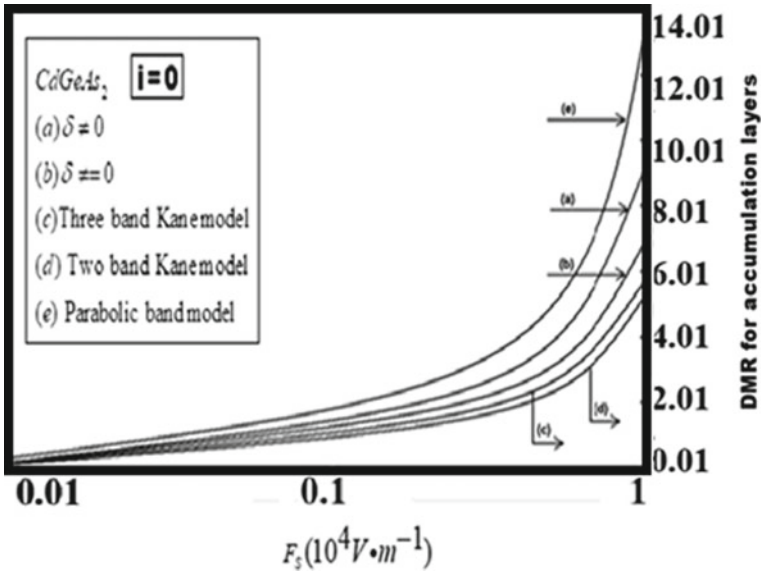

Fig. 7.2 The DMR in accumulation layers against for five cases of the indicated material

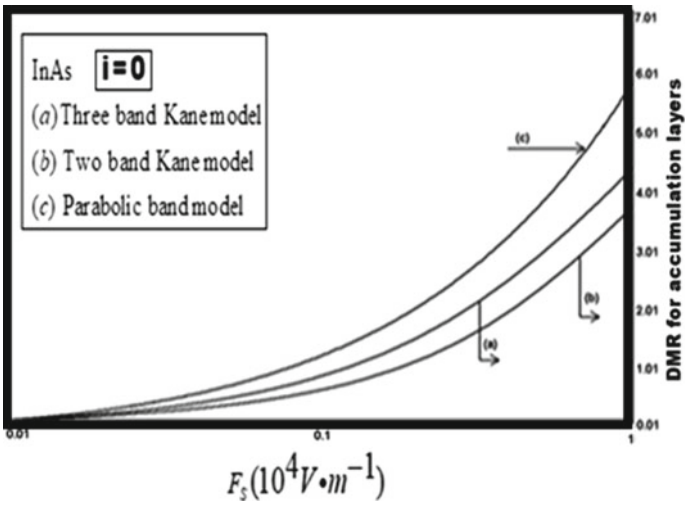

Fig. 7.3 The DMR in accumulation layers against F_s for three cases of the indicated material

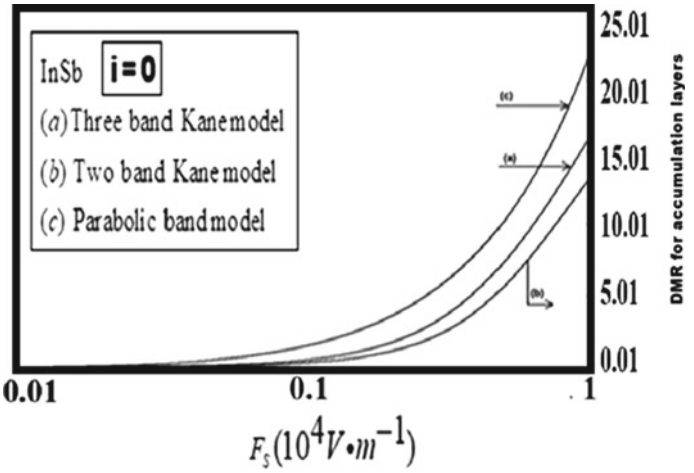

Fig. 7.4 The DMR in accumulation layers against F_s for three cases of the indicated material

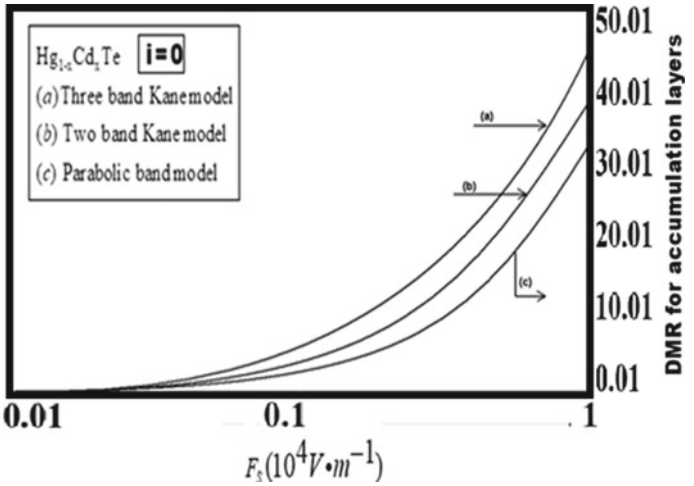

Fig. 7.5 The DMR in accumulation layers against F_s for three cases of the indicated material

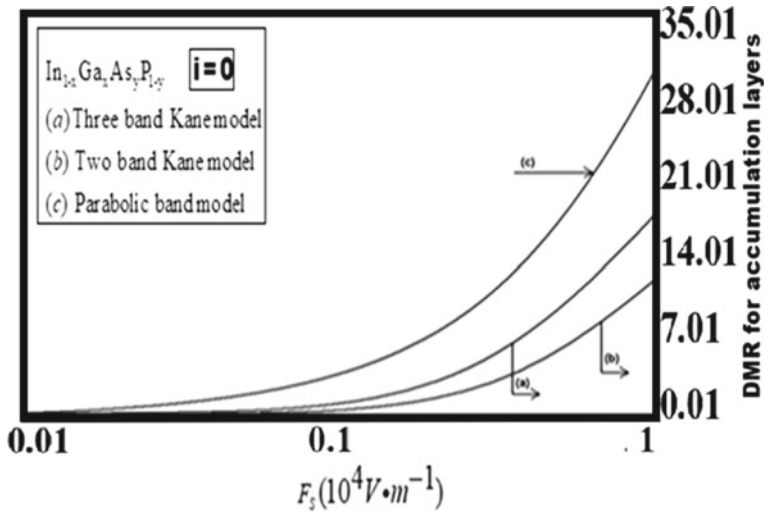

Fig. 7.6 The DMR in accumulation layers against F_s for three cases of the indicated material

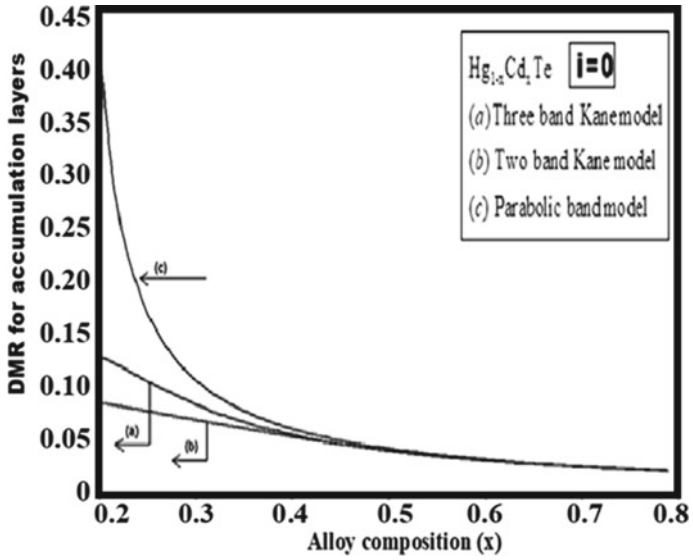

Fig. 7.7 The DMR in accumulation layers versus x for three cases of the indicated material

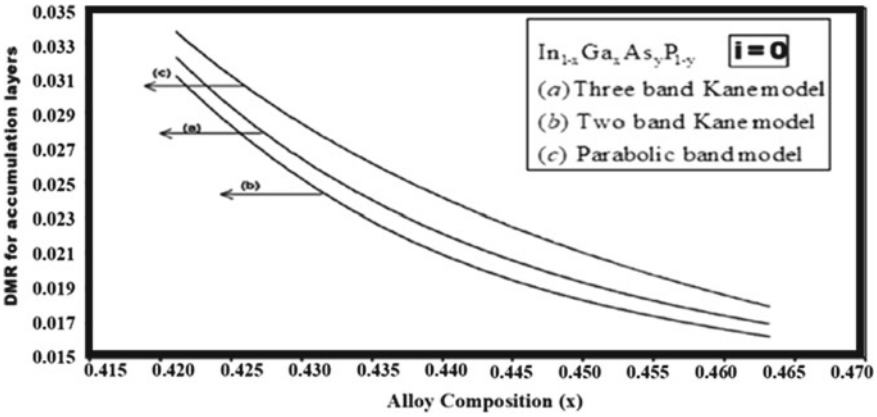

Fig. 7.8 The DMR in accumulation layers against x for three cases of the indicated material

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Chapter 8
The HUP and the ES in Heavily Doaped (HD) Non-parabolic Semiconductors Under Magnetic Quantization

8.1 Introduction

The magneto transport has been studied extensively for different compounds with varying energy band structures [1–14]. Here, we investigate the magneto ES in different compounds chronologically in accordance with the content of this chapter. In Sect. 8.3, we have studied a single ES dependent electronic property in this context.

8.2 Mathematical Basis

8.2.1 The Nonlinear Optical Semiconductors

The HDDR in this case is given by [14]

$$\frac{\hbar^2 k_z^2}{2m_{||}^*} = U_{1,\pm}(t_1) + iU_{2,\pm}(t_1) \tag{8.1}$$

where, $t_1 = (E, n, \eta_g)$

The EEM and n_0 become

$$m_{\pm}^*(t_2) = m_{||}^* U'_{1,\pm}(t_2) \tag{8.2}$$

$$n_0 = \text{R. } P. \overline{n}_0 \tag{8.3}$$

where

$$t_2 = (E_{FBHD}, n, \eta_g), \overline{n}_0 = (\sqrt{2m_{||}^*})\xi_7[\theta_{1,\pm}(t_2)] \tag{8.4}$$

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in which, $\xi_7 = \frac{eB}{2\pi^2\hbar^2}\sum_{n=0}^{n_{\max}}$

8.2.2 The III-V Semiconductors

(a) The HDDR becomes [14]

$$\frac{\hbar^2 k_z^2}{2m_c} = U_{3,\pm}(t_1) + iU_{4,\pm}(t_1) \tag{8.5}$$

The EEM and n_0 become

$$m_{\pm}^*(t_2) = m_{||}^* U'_{3,\pm}(t_2) \tag{8.6}$$

$$n_0 = R.P.\overline{n}_0 \tag{8.7}$$

where $\overline{n}_0 = (\sqrt{2m_{||}^*})\xi_7[\theta_{3,\pm}(t_2)]$

(b) **Two-Band Model of Kane**

The HDDR becomes [14]

$$\frac{\hbar^2 k_z^2}{2m_c} = \gamma_2(E,\eta_g) - (n+\frac{1}{2})\hbar\omega_0 \mp \frac{1}{2}g^*\mu_0 B = U_{5,\pm}(t_1) \tag{8.8}$$

The EEM and n_0 become

$$m^*(E_{FBHD},\eta_g) = m_c\gamma_2'(E_{FBHD},\eta_g) \tag{8.9}$$

$$n_0 = (\sqrt{2m_{||}^*})\xi_7[\theta_{5,\pm}(t_2)] \tag{8.10}$$

(c) **Parabolic Energy Bands**

The HDDR becomes [14]

$$\frac{\hbar^2 k_z^2}{2m_c} = \gamma_3(E,\eta_g) - (n+\frac{1}{2})\hbar\omega_0 \mp \frac{1}{2}g^*\mu_0 B = U_{6,\pm}(t_1) \tag{8.11}$$

The EEM and n_0 become

$$m^*(E_{FBHD},\eta_g) = m_c\gamma_3'(E_{FBHD},\eta_g) \tag{8.12}$$

$$n_0 = (\sqrt{2m^*_{||}})\xi_7[\theta_{7,\pm}(t_2)] \tag{8.13}$$

(d) **The Model of Stillman et al.**

The HDDR becomes [14]

$$k_z^2 = U_7(t_1) \tag{8.14}$$

The EEM and n_0 become

$$m^*(E_{FBHD}, \eta_g) = \frac{\hbar^2}{2} U_7'(t_2) \tag{8.15}$$

$$n_0 = 2\hbar\,\xi_7[\theta_9(t_2)] \tag{8.16}$$

(e) **The Model of Palik et al.**

The HDDR becomes [14]

$$k_z^2 = A_{35,\pm}(t_1) \tag{8.17}$$

The EEM and n_0 become

$$m^*_{\pm}(t_2) = \frac{\hbar^2}{2} A'_{35,\pm}(t_2) \tag{8.18}$$

$$n_0 = \hbar\xi_7[\theta_{11,\pm}(t_2)] \tag{8.19}$$

8.2.3 The II-VI Semiconductors

The HDDR becomes [14]

$$k_z^2 = U_{8,\pm}(t_1) \tag{8.20}$$

The EEM and n_0 become

$$m^*(E_{FBHD}, \eta_g) = \frac{\hbar^2}{2} U'_{8,\pm}(t_2) \tag{8.21}$$

$$n_0 = \hbar\,\xi_7[\theta_{13,\pm}(t_2)] \tag{8.22}$$

8.2.4 The IV-VI Semiconductors

(a) **Cohen Model**

The HDDR becomes [14]

$$\frac{\hbar^2 k_z^2}{2m_3} = U_{16,\pm}(t_1) \tag{8.23}$$

The EEM and n_0 become

$$m_{\pm}^{*}(t_2) = m_3 U'_{16,\pm}(t_2) \tag{8.24}$$

$$n_0 = (\sqrt{2m_3})\xi_7[\theta_{15,\pm}(t_2)] \tag{8.25}$$

where, $\theta_{15,\pm}(t_2) = [U_{16,\pm}(t_2)]^{1/2}$,

(b) **Lax Model**

The HDDR becomes [14]

$$\frac{\hbar^2 k_z^2}{2m_3} = U_{17,\pm}(t_1) \tag{8.26}$$

The EEM and n_0 become

$$m^{*}(E_{FBHD}, \eta_g) = m_3 U'_{17,\pm}(t_2) \tag{8.27}$$

$$n_0 = (\sqrt{2m_3})\xi_7[\theta_{17,\pm}(t_2)] \tag{8.28}$$

where, $\theta_{17,\pm}(t_2) = [U_{17,\pm}(t_2)]^{1/2}$,

(c) **Dimmock Model**

The HDDR becomes [14]

$$k_z^2 = U_{170}(t_1) \tag{8.29}$$

The EEM and n_0 become

$$m^{*}(t_2) = \frac{\hbar^2}{2} U'_{170}(t_2) \tag{8.30}$$

$$n_0 = 2\hbar\xi_7[\theta_{19}(t_2)] \tag{8.31}$$

where, $\theta_{19}(t_2) = [U_{170}(t_2)]^{1/2}$

(d) **Model of Bangert and Kastner**

The HDDR becomes [14]

$$k_z^2 = U_{18}(t_1) \tag{8.32}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2}\text{R. P.}\,[U_{18}(t_2)]' \tag{8.33}$$

$$n_0 = R.\,P.\,\overline{n}_0 \tag{8.34}$$

where

$$\overline{n}_0 = 2\hbar\xi_7[\theta_{21}(t_2)] \tag{8.35}$$

in which $\theta_{21}(t_2) = [U_{18}(t_2)]^{1/2}$

(e) **Model of Foley and Langenberg**

The HDDR becomes [14]

$$k_z^2 = U_{19}(t_1\tilde{)} \tag{8.36}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2}U'_{19}(t_2) \tag{8.37}$$

$$n_0 = 2\hbar\xi_7[\theta_{23}(t_2)] \tag{8.38}$$

where, $\theta_{23}(t_2) = [U_{19}(t_2)]^{1/2}$

8.2.5 The Stressed Kane-Type Semiconductors

The HDDR becomes [14]

$$(k_z')^2 = U_{41}(t_1) \tag{8.39}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{41}(t_2) \tag{8.40}$$

$$n_0 = 2\hbar\xi_7[\theta_{25}(t_2)] \tag{8.41}$$

where, $\theta_{25}(t_2) = [U_{41}(t_2)]^{1/2}$,

8.2.6 The Te

The HDDR becomes [14]

$$k_z^2 = U_{42,\pm}(t_1) \tag{8.42}$$

The EEM and n_0 become

$$m^*_{\pm}(t_2) = \frac{\hbar^2}{2} U'_{42,\pm}(t_2) \tag{8.43}$$

$$n_0 = \hbar\xi_7[\theta_{27,\pm}(t_2)] \tag{8.44}$$

where, $\theta_{27,\pm}(t_2) = [U_{42,\pm}(t_2)]^{1/2}$,

8.2.7 The Gallium Phosphide

The magneto HDDR becomes

$$k_z^2 = U_{43}(t_1) \tag{8.45}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{43}(t_2) \tag{8.46}$$

$$n_0 = 2\hbar\xi_7[\theta_{29}(t_2)] \tag{8.47}$$

where, $\theta_{29}(t_2) = [U_{43}(t_2)]^{1/2}$,

8.2.8 The Platinum Antimonide

The HDDR becomes

$$k_z^2 = U_{44}(t_1) \tag{8.48}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{44}(t_2) \tag{8.49}$$

$$n_0 = 2\hbar\xi_7[\theta_{31}(t_2)] \tag{8.50}$$

where, $\theta_{31}(t_2) = [U_{44}(t_2)]^{1/2}$,

8.2.9 The Bismuth Telluride

The HDDR becomes

$$k_x^2 = U_{45}(t_1) \tag{8.51}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{45}(t_2) \tag{8.52}$$

$$n_0 = 2\hbar\xi_7[\theta_{33}(t_2)] \tag{8.53}$$

where, $\theta_{33}(t_2) = [U_{45}(t_2)]^{1/2}$

8.2.10 The Germanium

a. **Model of Cardona et al.**

The HDDR becomes

$$k_x^2 = U_{46}(t_1) \tag{8.54}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{46}(t_2) \tag{8.55}$$

$$n_0 = 2\hbar\xi_7[\theta_{35}(t_2)] \tag{8.56}$$

where, $\theta_{35}(t_2) = [U_{46}(t_2)]^{1/2}$

b. **Model of Wang and Ressler**

The HDDR becomes

$$k_x^2 = U_{47}(t_1) \tag{8.57}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{47}(t_2) \tag{8.58}$$

$$n_0 = 2\hbar\xi_7[\theta_{37}(t_2)] \tag{8.59}$$

where, $\theta_{37}(t_2) = [U_{47}(t_2)]^{1/2}$

8.2.11 The Gallium Antimonide

The magneto HDDR becomes

$$k_z^2 = U_{48}(t_1) \tag{8.60}$$

The EEM and n_0 become

$$m^*(t_2) = \frac{\hbar^2}{2} U'_{48}(t_2) \tag{8.61}$$

$$n_0 = 2\hbar\xi_7[\theta_{39}(t_2)] \tag{8.62}$$

where, $\theta_{39}(t_2) = [U_{48}(t_2)]^{1/2}$

8.2.12 The II-V Materials

The magneto HDDR becomes

$$k_y^2 = U_{49,\pm}(t_1) \tag{8.63}$$

The EEM and n_0 become

$$m_{\pm}^*(t_2) = \frac{\hbar^2}{2} U'_{49,\pm}(t_2) \tag{8.64}$$

$$n_0 = \hbar \xi_7 [\theta_{41,\pm}(t_2)] \tag{8.65}$$

where, $\theta_{41,\pm}(t_2) = [U_{49,\pm}(t_2)]^{1/2}$

8.2.13 The Lead Germanium Telluride

The magneto DR in HD $Pb_{1-x}Ge_xTe$ becomes

$$k_z^2 = U_{50,\pm}(t_1) \tag{8.66}$$

The EEM and n_0 become

$$m_{\mp}^*(t_2) = \frac{\hbar^2}{2} U'_{50,\mp}(t_2) \tag{8.67}$$

$$n_0 = \hbar \xi_7 [\theta_{43,\pm}(t_2)] \tag{8.68}$$

where, $\theta_{43,\pm}(t_2) = [U_{50,\pm}(t_2)]^{1/2}$

8.3 Result and Discussions

In this chapter, we have plotted the normalized DMR (ϕ_1) with different physical variables in Figs. 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10, 8.11 and 8.12 to illustrate at least a single electronic property dependent on ES in this context. From the said figures, we note the following points:

1. From Figs. 8.1, 8.2, 8.4, 8.7, 8.8 and 8.9, we note that the ϕ_1 'oscillates with (B^{-1}) due to SdH effect.

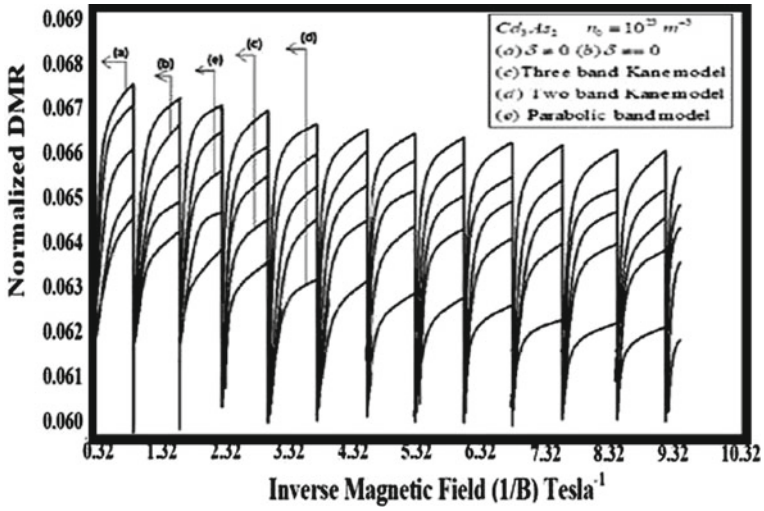

Fig. 8.1 The ϕ_1 against $(1/B)$ for indicated HD material

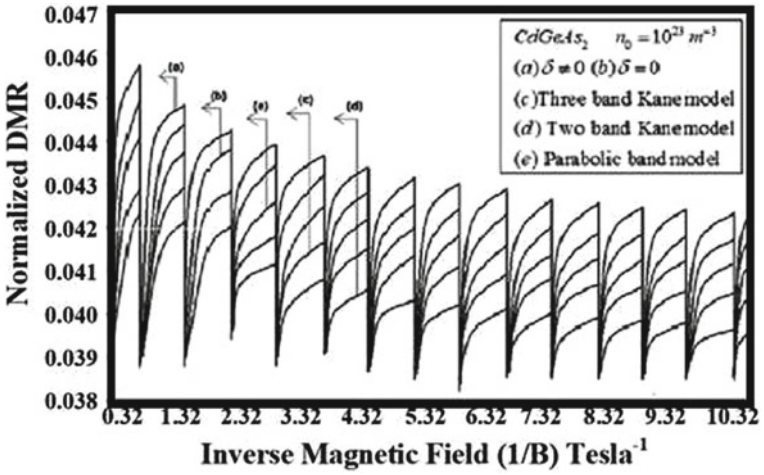

Fig. 8.2 The ϕ_1 against $(1/B)$ for indicated HD material

2. From Figs. 8.3, 8.5, 8.6, 8.10, 8.11 and 8.12, we observe that the ϕ_1 oscillates with n_0.

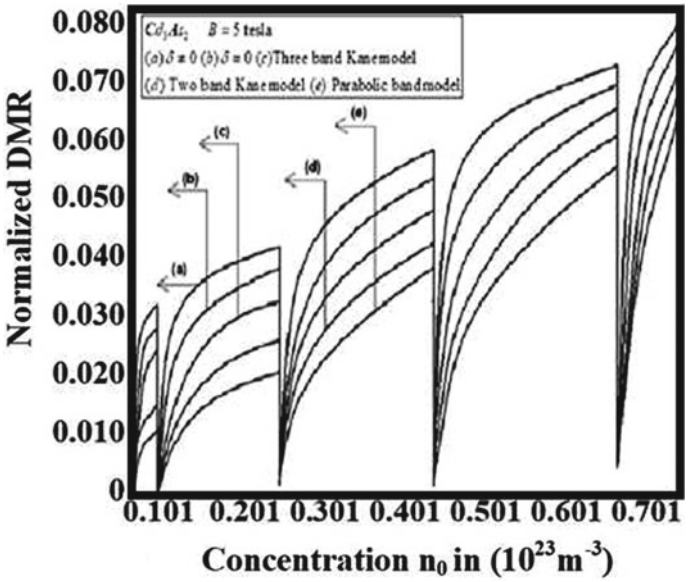

Fig. 8.3 The ϕ_1 against versus n_0 for indicated HD material

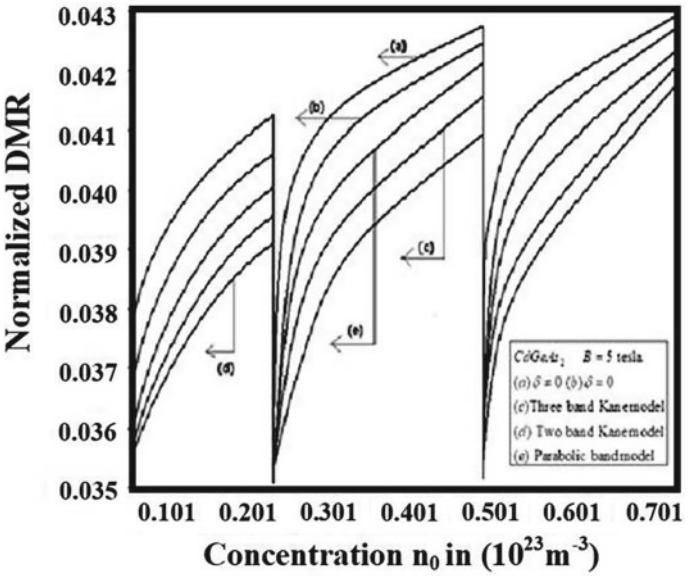

Fig. 8.4 The ϕ_1 against versus n_0 for indicated HD material

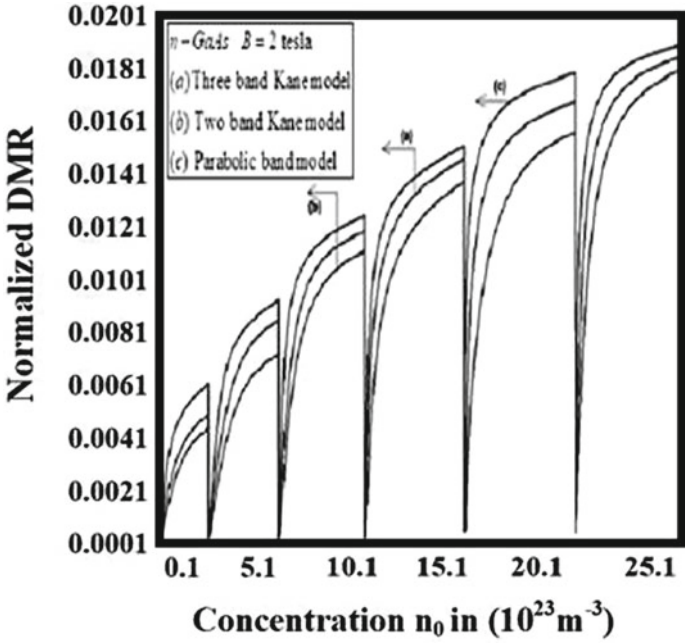

Fig. 8.5 The ϕ_1 against n_0 for indicated HD material

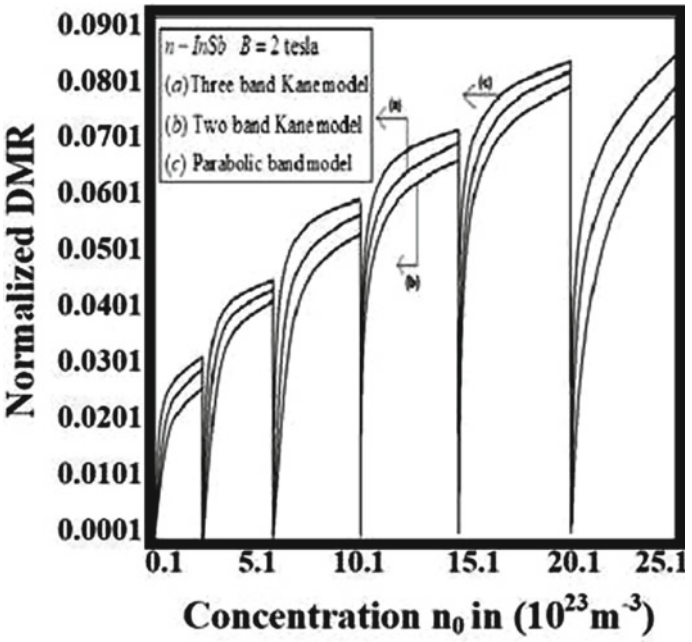

Fig. 8.6 The ϕ_1 against n_0 for indicated HD material

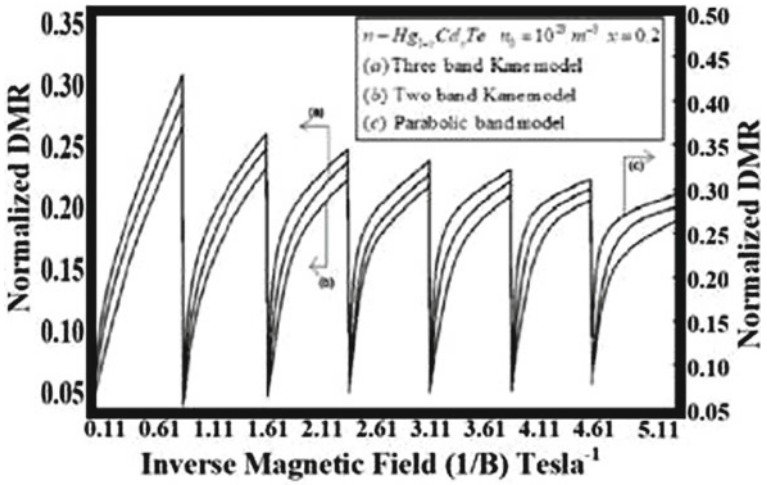

Fig. 8.7 The ϕ_1 against $(1/B)$ for indicated HD material

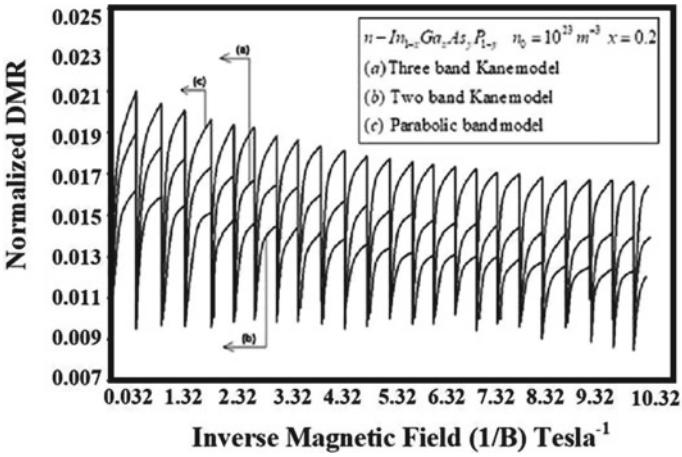

Fig. 8.8 The ϕ_1 against $(1/B)$ for indicated HD material

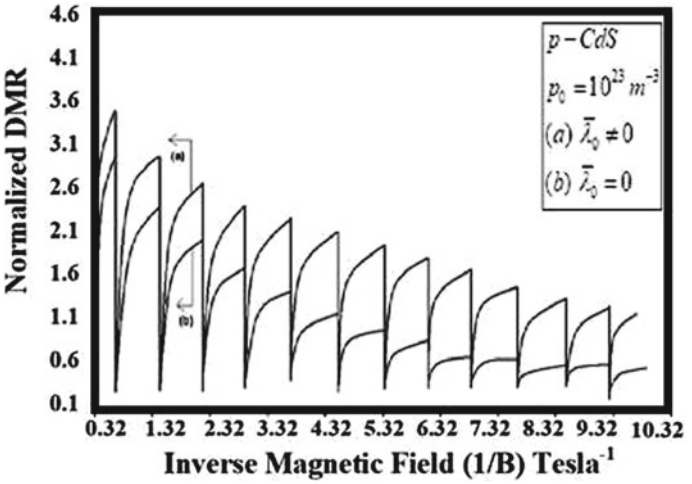

Fig. 8.9 The ϕ_1 against $(1/B)$ for indicated HD material

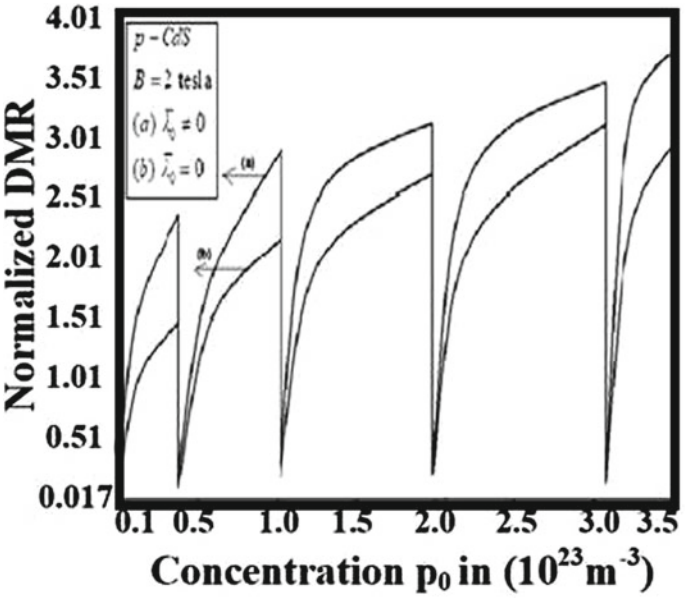

Fig. 8.10 The ϕ_1 against p_0 for indicated HD material

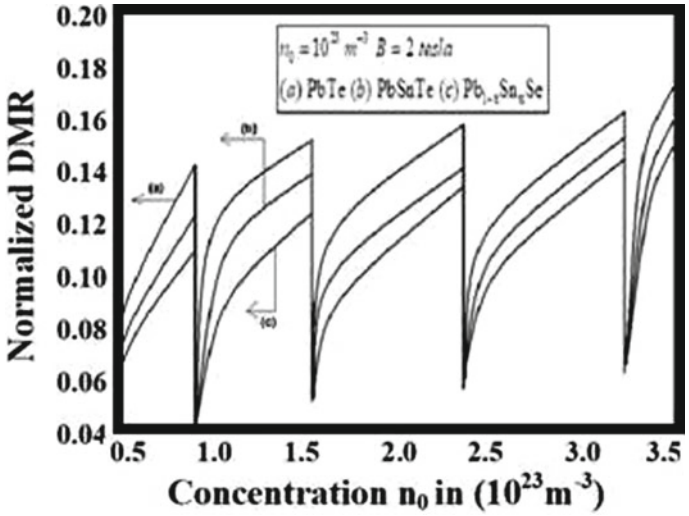

Fig. 8.11 The ϕ_1 against n_0 for indicated HD materials

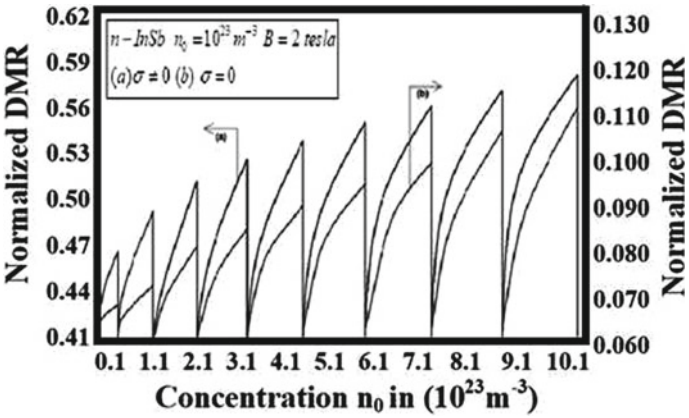

Fig. 8.12 The ϕ_1 against n_0 for indicated HD material

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Chapter 9
The HUP and the ES in HD Non-parabolic Semiconductors Under Cross-Fields Configuration

9.1 Introduction

The influence of cross fields on the electronic properties of different compounds has extensively been studied [1–3, 3–5, 5–12]. Here, we investigate the ES under the said external conditions for different semiconductors chronologically in accordance with the content of this chapter. Section 9.3 investigates one ES dependent electronic property namely the DMR.

9.2 Mathematical Basis

9.2.1 The Nonlinear Optical Semiconductors

The HDDR becomes [13]

$$T_{22}(t_3) = \left[\left(\left(n+\frac{1}{2}\right)\hbar\omega_{01}\right)\right] + \left(\frac{[\hbar k_z(E)]^2}{2a(t_3)}\right) - \left(\frac{E_0\hbar k_y\rho(t_3)}{B}\right) - \left(\frac{M_\perp\rho^2(t_3)E_o^2}{2B^2}\right) \tag{9.1}$$

where, $t_3 = (E, \eta_g)$ and $\omega_{01} = \frac{|e|B}{m_\perp^*}$

The EEM's and n_0 become

$$m_z^*(t_4) = R.P\left[a'(t_5)\left[T_{22}(t_5) - \left(n+\frac{1}{2}\right)\hbar\omega_{01} + \frac{M_\perp\rho^2(t_5)E_0^2}{2B^2}\right] + a(t_5)\left[T_{22}'(t_5) + \frac{M_\perp\rho(t_5)\rho'(t_5)E_0^2}{B^2}\right]\right] \tag{9.2}$$

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and

$$m_y^*(t_4) = \left(\frac{B}{E_0}\right)^2 R.\,P\left[\rho(t_5)^{-3}\right]\left[T_{22}(t_5) - \left(n+\frac{1}{2}\right)\hbar\omega_{01} + \frac{M_\perp \rho^2(t_5)E_0^2}{2B^2}\right]$$
$$\left[\left[\rho(t_5)\left[T'_{22}(t_5) + \frac{M_\perp \rho(t_5)\rho'(t_5)E_0^2}{B^2}\right]\right.\right.$$
$$\left.\left.-\left[T_{22}(t_5) - (n+\frac{1}{2})\hbar\omega_0 + \frac{M_\perp \rho^2(t_5)E_0^2}{2B^2}\right]\rho'(t_5)\right]\right] \tag{9.3}$$

$$n_0 = R.P.\overline{n}_0 \tag{9.4}$$

where, $t_4 = (\overline{E}_{\mathrm{FBDH},\eta_g}, n, E_0)$, $t_5 = (\overline{E}_{\mathrm{FBDH},\eta_g})$, $\overline{E}_{\mathrm{FBHD}}$ is the corresponding Fermi energy

and

$$\overline{n}_0 = \left(\frac{2g_v B\sqrt{2}}{3L_x\pi^2\hbar^2 E_0}\right)\sum_{n=0}^{n_{\max}}\left[T_{41\mathrm{HD}}(n,t_5)\right] \tag{9.5}$$

9.2.2 The Kane-Type III-V Semiconductors

(a) With the constraints $\delta = 0, \Delta_{||} = \Delta_\perp = \Delta$ and $m_{||}^* = m_\perp^* = m_{c,}$, (9.1) becomes

$$T_{33}(t_3) = \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{[\hbar k_z(E)]^2}{2m_c} - \frac{E_0}{B}\hbar k_y\{T_{33}(t_3)\}'$$
$$-\frac{m_c E_0^2[\{T_{33}(t_3)'\}]^2}{2B^2} \tag{9.6}$$

The EEM s' n_0 become

$$m_z^*(t_4) = m_c\ R.\ P.\left[\{T_{33}(t_5)\}'' + \frac{m_c E_0^2\{T_{33}(t_5)'\}\{T_{33}(t_5)\}''}{B^2}\right] \tag{9.7}$$

$$m_y^*(t_4) = \left(\frac{B}{E_0}\right)^2 R.\,P.\left[\{T_{33}(t_5)\}'\right]^{-1}$$
$$+\left[T_{33}(t_5) - \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{m_c E_0^2[\{T_{33}(t_5)\}'']^2}{2B^2}\right]$$

$$\left[\frac{-\{T_{33}(t_5)\}''}{[\{T_{33}(t_5)\}']^2}\left[T_{33}(t_5)-\left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{m_c E_0^2[\{T_{33}(t_5)\}']}{2B^2}\right]+1\frac{m_c E_0^2\{T_{33}(t_5)\}''}{B^2}\right] \tag{9.8}$$

and

$$n_0 = R.P.\ \overline{n}_0 \tag{9.9}$$

where,

$$\overline{n}_0 = \frac{2g_v B\sqrt{2m_c}}{3L_x\pi^2\hbar^2 E_0}\sum_{n=0}^{n_{\max}}[T_{43\mathrm{HD}}(n,t_5)] \tag{9.10}$$

(b) Under the condition $\Delta >> E_g$, (9.6) becomes

$$\gamma_2(t_3) = \left(n+\frac{1}{2}\right)\hbar\omega_0 - \frac{E_0}{B}\hbar k_y\gamma_2'(t_3) - \frac{m_c E_0^2}{2B^2}\gamma_2'(t_3) + \frac{[\hbar k_z(E)]^2}{2m_c} \tag{9.11}$$

The EEM s' and n_0 become

$$m_z^*(t_4) = m_c\left[\{\gamma_2(t_5)\}'' + \frac{m_c E_o^2\{\gamma_2(t_5)\}'\{\gamma_2(t_5)\}''}{B^2}\right] \tag{9.12}$$

$$m_y^*(t_4) = \left(\frac{B}{E_0}\right)^2\frac{1}{[\{\gamma_2(t_5)\}']}\left[\gamma_2(t_5)-\left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{m_c E_o^2[\{\gamma_2(t_5)\}']^2}{2B^2}\right] \left[\frac{-\{\gamma_2(t_5)\}''}{[\{\gamma_2(t_5)\}']^2}\left[\gamma_2(t_5)-\left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{m_c E_0^2[\{\gamma_2(t_5)\}']^2}{2B^2}\right]+1+\frac{m_c E_0^2\{\gamma_2(t_5)\}''}{B^2}\right] \tag{9.13}$$

and

$$n_0 = \frac{2g_v B\sqrt{2m_c}}{3L_x\pi^2\hbar^2 E_0}\sum_{n=0}^{n_{\max}}[T_{47\mathrm{HD}}(n,t_5)] \tag{9.14}$$

(c) (c). For $a \to 0$ become,

$$\gamma_3(t_3) = \left(n+\frac{1}{2}\right)\hbar\omega_0 - \frac{E_0}{B}\hbar k_y\gamma_3'(t_3) - \frac{m_c E_0^2}{2B^2}\left(\gamma_3'(t_3)\right)^2 + \frac{[\hbar k_z(E)]^2}{2m_c} \tag{9.15}$$

The use of (9.15) leads to the expressions of the EEMs' along z and y directions and n_0 as

$$m_z^*(t_4) = m_c\left[\left\{\gamma_3(t_5)'' + \frac{m_c E_0^2\{\gamma_3(t_5)\}'\{\gamma_3(t_5)\}''}{B^2}\right\}\right] \tag{9.16}$$

$$\begin{aligned} m_y^*(t_4) = \left(\frac{B}{E_0}\right)^2 \frac{1}{[\{\gamma_3(t_5)\}']}\Bigg[\gamma_3(t_5) - \left(n+\frac{1}{2}\right)\hbar\omega_0 \\ + \frac{m_c E_0^2[\{\gamma_3(t_5)\}']^2}{2B^2}\Bigg]\Bigg[\frac{-\{\gamma_3(t_5)\}''}{[\{\gamma_3(t_5)\}']^2}\Big[\gamma_3(t_5) \\ - \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{m_c E_0^2[\{\gamma_3(t_5)\}']^2}{2B^2}\Big] + 1 \\ + \frac{m_c E_0^2\{\gamma_3(t_5)\}''}{B^2}\Bigg] \end{aligned} \tag{9.17}$$

and

$$n_0 = \frac{2g_v B\sqrt{2m_c}}{3L_x\pi^2\hbar^2 E_0}\sum_{n=0}^{n_{\max}}[T_{49\mathrm{HD}}(n, t_5)] \tag{9.18}$$

9.2.3 The II-VI Semiconductors

The HDDR becomes [13]

$$\begin{aligned} \gamma_3(t_3) = \beta_1(n, E_0) - \frac{E_0}{B}\hbar k_y\gamma_3'(t_3) \\ - \frac{m_{||}^* E_0^2}{2B^2}\left(\gamma_3'(t_3)\right)^2 + \frac{[\hbar k_z(E)]^2}{2m_{||}^*} \end{aligned} \tag{9.19}$$

The EEMs' n_0 become

$$m_z^*(t_4) = m_{||}^*\left[\{\gamma_3(t_5)\}'' + \frac{m_{||}^* E_0^2\{\gamma_3(t_5)\}'\{\gamma_3(t_5)\}'\prime}{B^2}\right] \tag{9.20}$$

$$\begin{aligned} m_y^*(t_4) = \left(\frac{B}{E_0}\right)^2 \frac{1}{[\{\gamma_3(t_5)\}']}\Big[\{\gamma_3(t_5)\} \\ -\beta_1(n, E_0) + \frac{m_{||}^* E_0^2[\{\gamma_3(t_5)\}']^2}{2B^2}\Bigg] \end{aligned}$$

$$\left[\frac{-\{\gamma_3(t_5)\}''}{[-\{\gamma_3(t_5)\}']^2}\left[\gamma_3(t_5)-\beta_1(n,E_0)+\frac{m_{||}^* E_0^2[\{\gamma_3(t_5)\}']^2}{2B^2}\right]+1+\frac{m_{||}^* E_0^2[\{\gamma_3(t_5)\}']}{B^2}\right] \tag{9.21}$$

and

$$n_0=\frac{2g_v B\sqrt{2m_c}}{3L_x\pi^2\hbar^2E_0}\sum_{n=0}^{n_{\max}}[T_{53\mathrm{HD}}(n,t_5)] \tag{9.22}$$

9.2.4 The IV-VI Semiconductors

The HDDR becomes [13]

$$g^*(t_3)=\left(n+\frac{1}{2}\right)\hbar\overline{\omega}_{i1}(t_3)+\frac{\hbar^2k_z^2}{2M_3^*(t_3)}-\frac{E_0}{B}\rho_1^*(t_3)\hbar k_y-\frac{E_0^2}{2B^2}\left[\rho_1^*(t_3)\right]^2M_1^*(t_3) \tag{9.23}$$

The EEM along y direction and n_0 become

$$\begin{aligned}m_y^*(t_4)=(B/E_0)^2&\text{Real part of}\left[\rho_1^*(t_5)\right]^{-3}\\&\left[g^*(t_5)-\left(n+\frac{1}{2}\right)\hbar\overline{\omega}_{i1}(t_5)\right.\\&+\left.\frac{E_0^2}{2B^2}\left[\rho_1^*(t_5)\right]^2M_1^*(t_5)\right]\left[\rho_1^*(t_5)\right]\left[\left[g^*(t_5)\right]'\right.\\&-\left(n+\frac{1}{2}\right)\hbar[\overline{\omega}_i(t_5)]'\\&+\frac{E_0^2}{2B^2}\left[[\rho_1^*(t_5)^2[M_1^*(t_5)]'\right]-\left[\rho_1^*(t_5)\right]'\left[g^*(t_5)\right.\\&-\left.\left.\left(n+\frac{1}{2}\right)\hbar\overline{\omega}_{i1}(t_5)\frac{E_0^2}{2B^2}\left[\rho_1^*(t_5)\right]^2M_1^*(t_5)\right]\right]\end{aligned} \tag{9.24}$$

and

$$n_0=R.P.\overline{n}_0 \tag{9.25}$$

where

$$\overline{n}_0 = \frac{2B}{3L_x\pi^2\hbar^2 E_0}\sum_{n=0}^{n_{\max}}[T_{4131\mathrm{HD}}(n,t_5)] \tag{9.26}$$

9.2.5 The Stressed Kane-Type Semiconductors

The HDDR becomes [13]

$$G^*(t_3) = \left(n+\frac{1}{2}\right)\hbar\overline{\omega}_i(t_3) + \frac{\hbar^2 k_z^2}{2m_3^*(t_3)} - \frac{E_0}{B}\rho^*(t_3)\left[\frac{m_1^*(t_3)}{m_2^*(t_3)}\right]^{1/2}\hbar k_y - \frac{E_0^2}{2B^2}\left(\rho^*(t_3)\right)^2 m_1^*(t_3) \tag{9.27}$$

The EEM along z direction and n_0 become

$$\begin{aligned} m_z^*(t_4) = \Bigg[\left[m_3^*(t_5)\right]'\Bigg[G^*(t_5) - \left(n+\frac{1}{2}\right)\hbar\overline{\omega}_i(t_5) \\ + \frac{E_0^2}{2B^2}\left[\rho^*(t_5)^2 m_1^*(t_5)\right] + \left[m_3^*(t_5)\right]\Big[\left[G^*(t_5)\right]' \\ - \left(n+\frac{1}{2}\right)\hbar[\overline{\omega}_i(t_5)]' \\ + \frac{E_0^2}{2B^2}[2\left[\rho^*(t_5)\left[\rho^*(t_5)'\right]\left[m_1^*(t_5)\right]\right] \\ +\left[m_1^*(t_5)\right]'\left[\rho^*(t_5)\right]^2\Big]\Big]\Bigg] \end{aligned} \tag{9.28}$$

$$\begin{aligned} m_y^*(t_4) = (B/E_0)^2\left[m_4^*(t_5)\right]^{-3}\Bigg[G^*(t_5) - \left(n+\frac{1}{2}\right)\hbar\overline{\omega}_i(t_5) \\ + \frac{E_0^2}{2B^2}\left[\rho^*(t_5)^2 m_1^*(t_5)\right]\left[\left[m_4^*(t_5)\right]\left[\left[G^*(t_5)\right]'\right.\right. \\ - \left(n+\frac{1}{2}\right)\hbar[\overline{\omega}_i(t_5)]' \\ + \frac{E_0^2}{2B^2}\Big[\left[\rho^*(t_5)^2 m_1^*(t_5)\right]'\Big]\left[m_4^*(t_5)\right]'\left[G^*(t_5)\right] \\ -\left(n+\frac{1}{2}\right)\hbar\overline{\omega}_i(t_5) + \frac{E_0^2}{2B^2}\left[\rho^*(t_5)^2 m_1^*(t_5)\right]\Bigg] \end{aligned} \tag{9.29}$$

and

$$n_0 = \frac{2B}{3L_x\pi^2\hbar^2 E_0}\sum_{n=0}^{n_{\max}}[T_{413HD}(n,t_5)] \tag{9.30}$$

9.3 Result and Discussion

We have plotted the normalized DMR (ϕ_1) in Figs. 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14, 9.15, 9.16, 9.17 and 9.18 with different physical variables to illustrate at least a single electronic property dependent on ES in this context. From the said figures, we note the following points:

1. From Figs. 9.4, 9.5 and 9.9, we note that the ϕ_1 oscillates with $(1/B)$ due to SdH effect.
2. Figures 9.1, 9.2, 9.6, 9.10, 9.11, 9.14, 9.15, 9.16, 9.17 and 9.18 illustrate that the ϕ_1 oscillates with increasing carrier density.
3. From Fig. 9.13, we note that the ϕ_1 decreases with increasing x with different numerical values.
4. From Figs. 9.3, 9.7, 9.8 and 9.12, we note that the E_0 enhances the ϕ_1 with different numerical values.

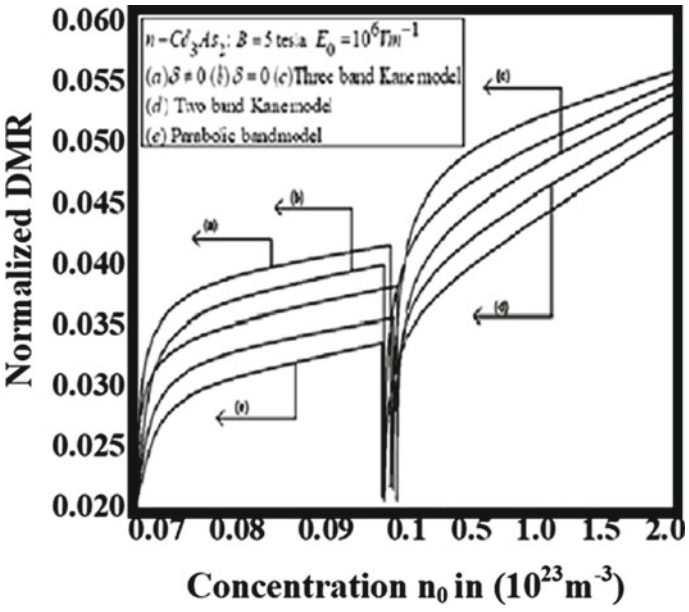

Fig. 9.1 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

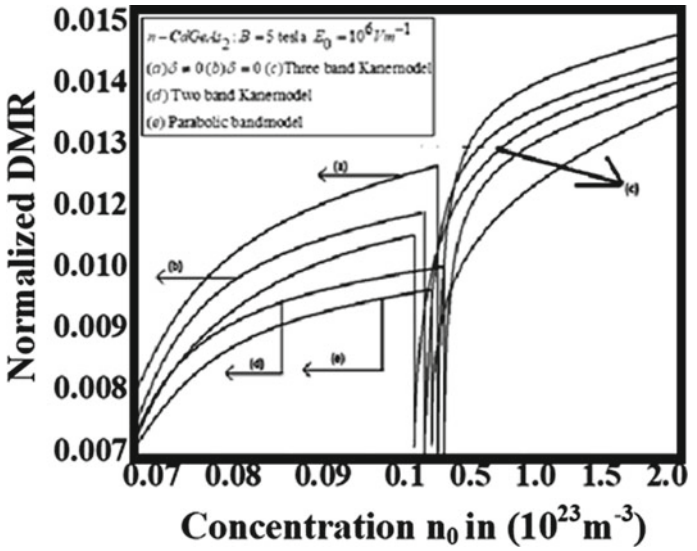

Fig. 9.2 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

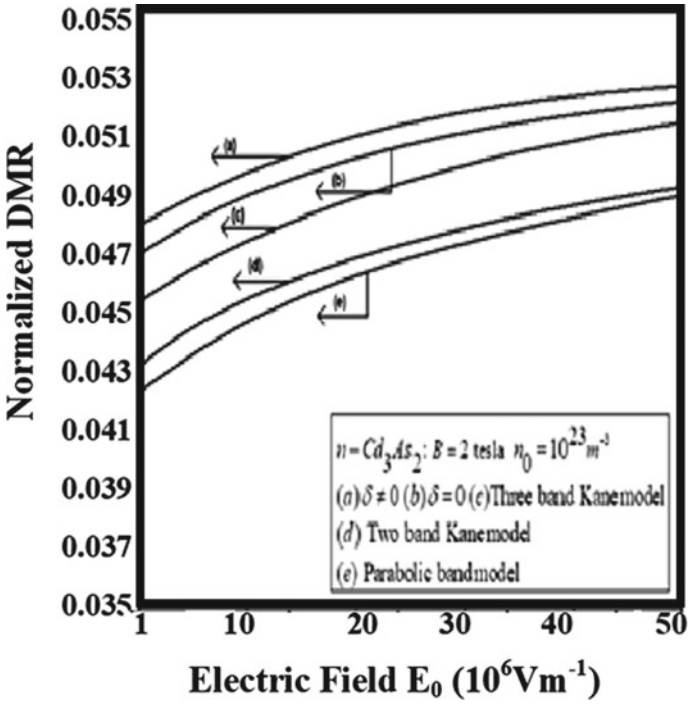

Fig. 9.3 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

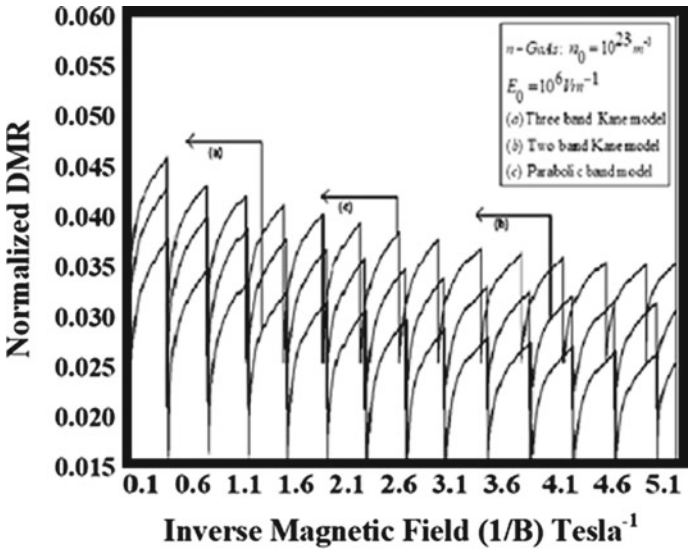

Fig. 9.4 The ϕ_1 against $(1/B)$ for the indicated HD material for $E_0 \perp B$

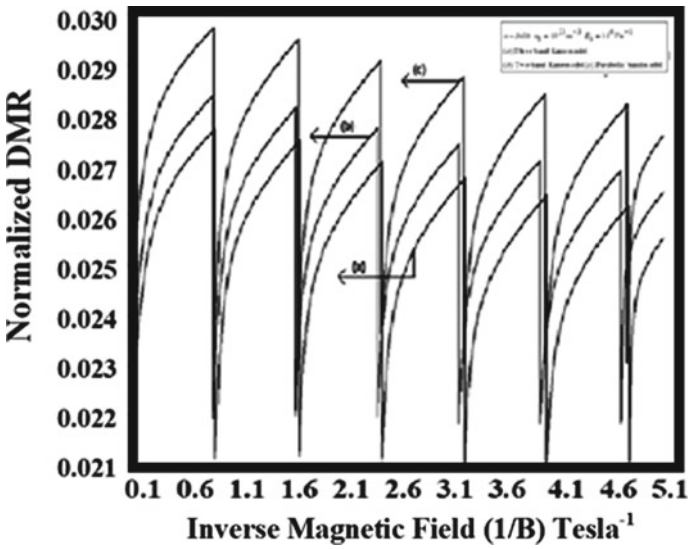

Fig. 9.5 The ϕ_1 against $(1/B)$ for the indicated HD material for $E_0 \perp B$

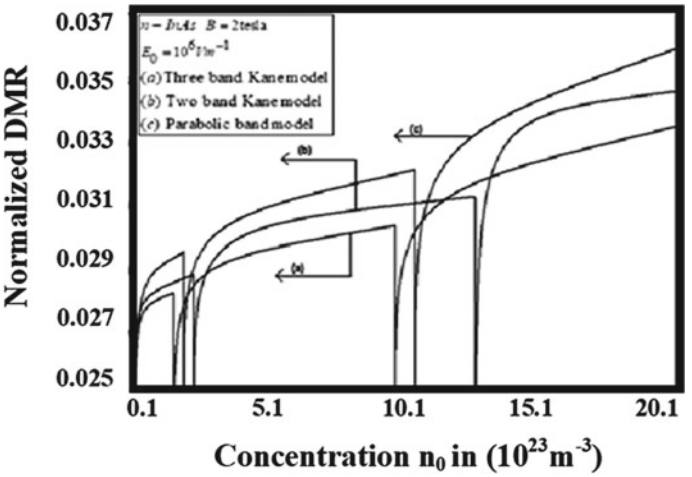

Fig. 9.6 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

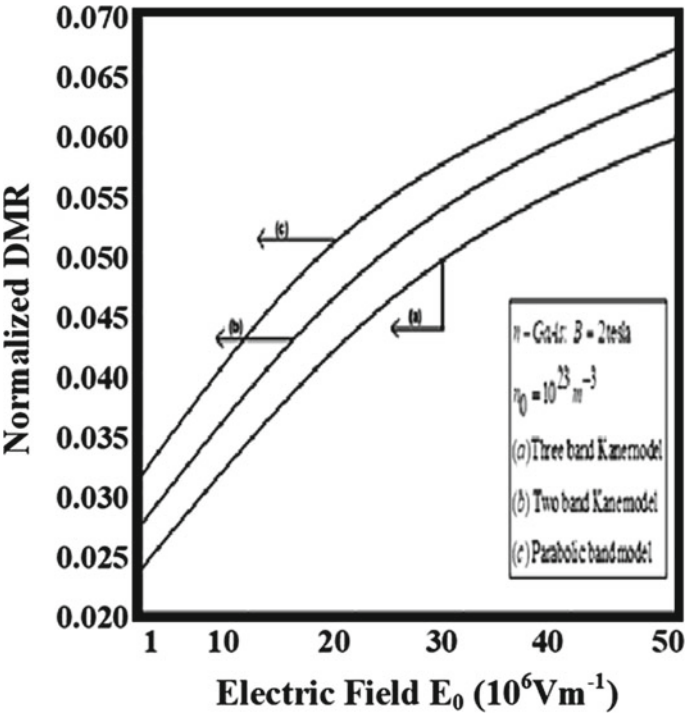

Fig. 9.7 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

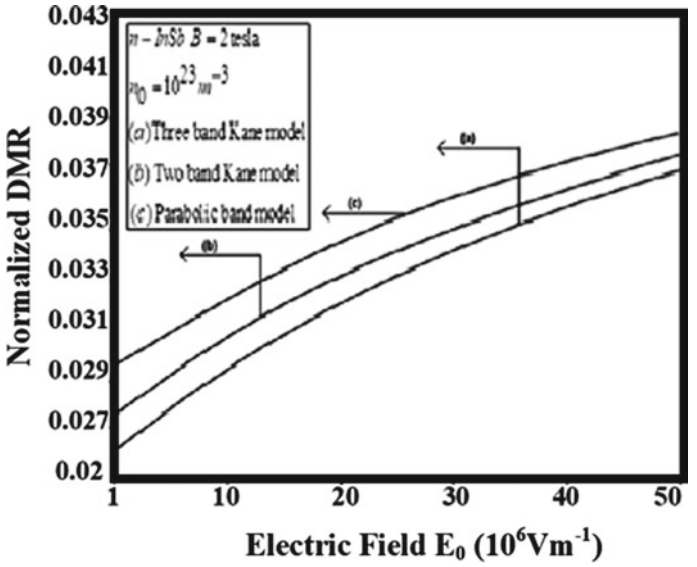

Fig. 9.8 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

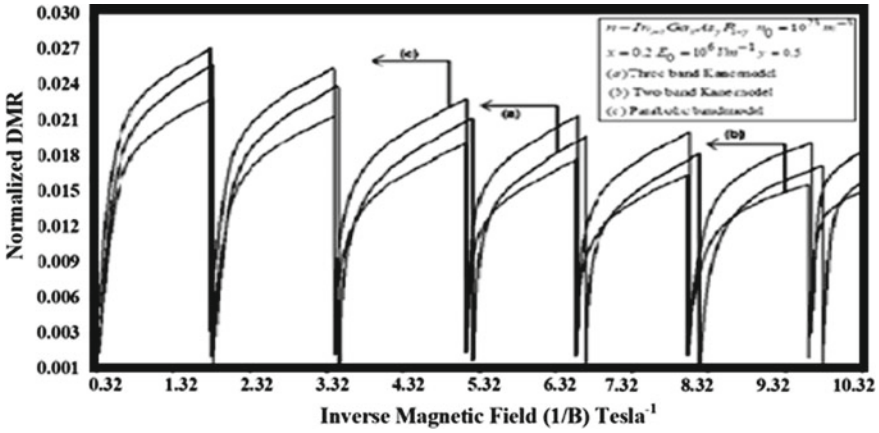

Fig. 9.9 The ϕ_1 against $(1/B)$ for the indicated HD material for $E_0 \perp B$

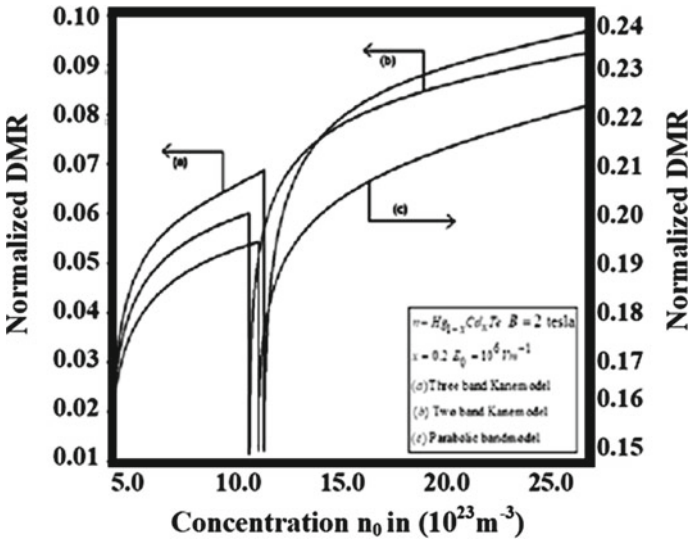

Fig. 9.10 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

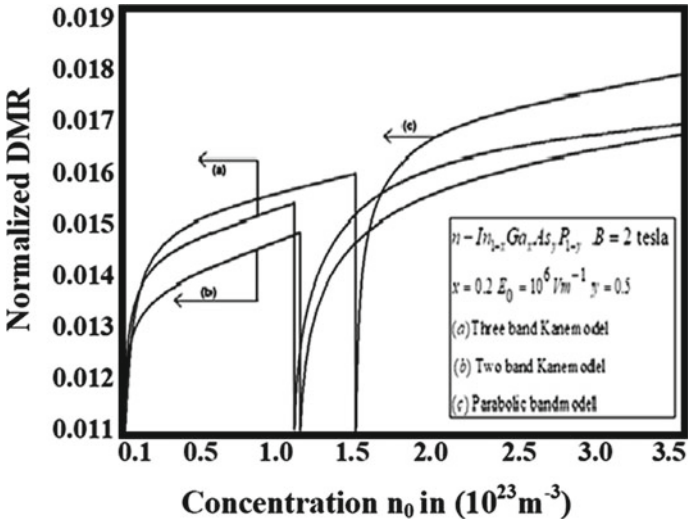

Fig. 9.11 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

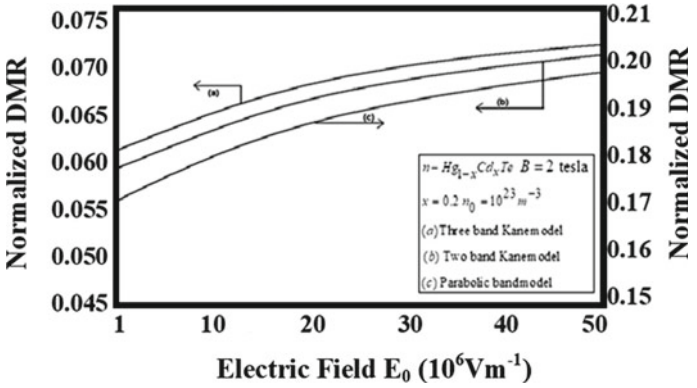

Fig. 9.12 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

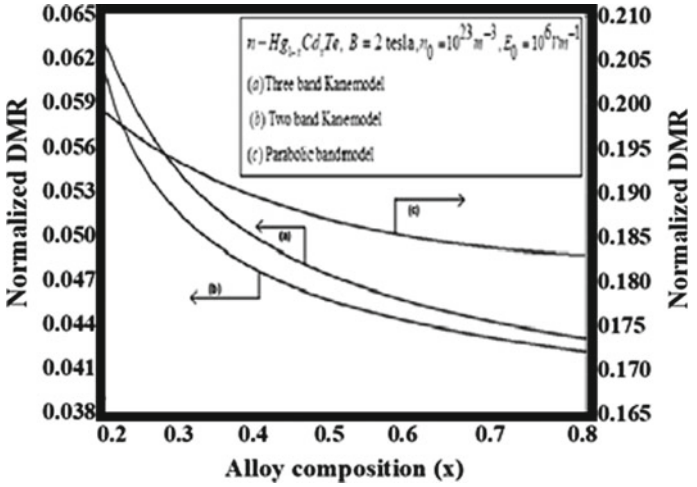

Fig. 9.13 The ϕ_1 against (x) for the indicated HD material for $E_0 \perp B$

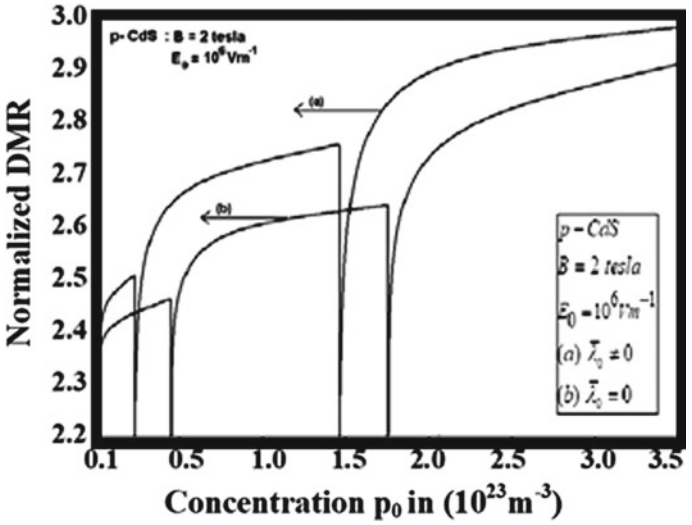

Fig. 9.14 The ϕ_1 against (p_0) for the indicated HD material for $E_0 \perp B$

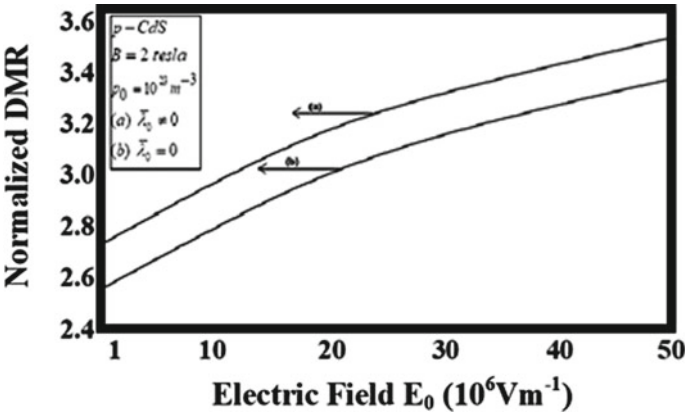

Fig. 9.15 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

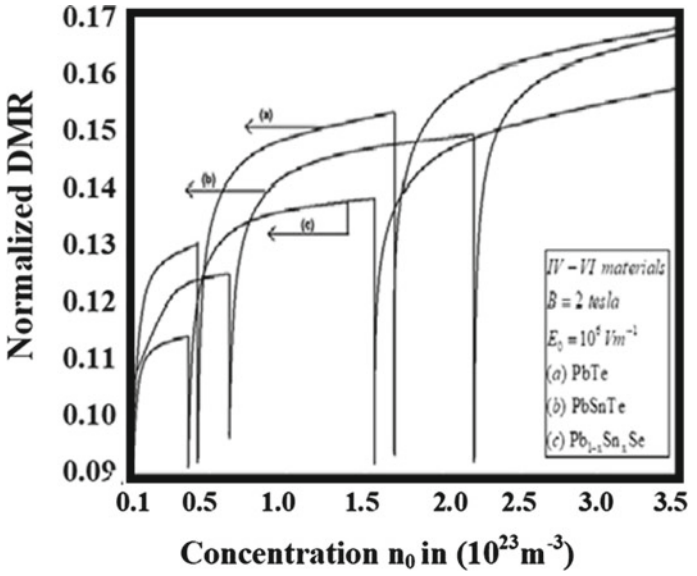

Fig. 9.16 The ϕ_1 against (n_0) for the indicated HD material for $E_0 \perp B$

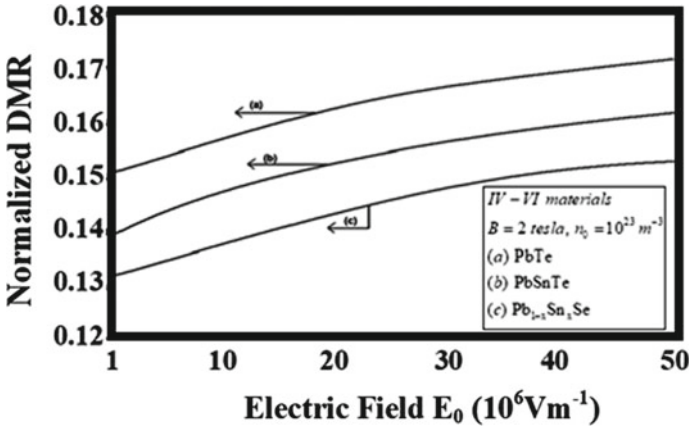

Fig. 9.17 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

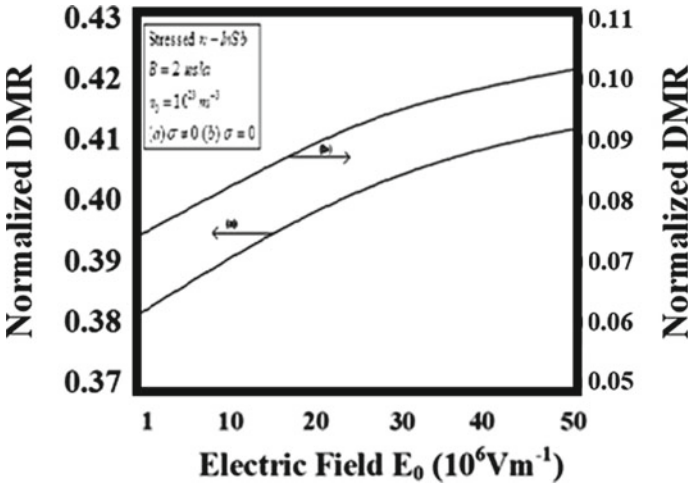

Fig. 9.18 The ϕ_1 against (E_0) for the indicated HD material for $E_0 \perp B$

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Chapter 10
The HUP and the ES in Heavily Doped (HD) Non-parabolic Semiconductors Under Magneto-Size Quantization

10.1 Introduction

The literature of the electron transport of QW compounds under magnetic quantization is rather large [1–35]. In this chapter, we investigate the ES in size-quantized films in the presence of magnetic field in Sect. 10.2, and the summary and conclusion of this chapter have been given in Sect. 10.3.

10.2 Mathematical Basis

10.2.1 The Nonlinear Optical Semiconductors

The HDDR is given by [35]

$$\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_{||}^*} = U_{1,\pm}(e_{81}, n, \eta_g) + iU_{2,\pm}(e_{81}, n, \eta_g) \tag{10.1}$$

The ES becomes

$$\overline{n}_{02D} = \zeta_1[1 + \exp(-\eta_{91})]^{-1} \tag{10.2}$$

where, $\overline{n}_{02D} = \zeta_1[1 + \exp(-\eta_{91})]$ $\zeta_1 = \frac{g_v eB}{2\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{n_z=1}^{n_{z\max}}$, $\eta_{91} = [E_{F91HD2D} - e_{91}](k_BT)^{-1}$ and

$E_{F91HD2D}$ is the corresponding Fermi energy

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10.2.2 The III-V Semiconductors

(a) **Three-Band Model of Kane**

The HDDR becomes [35]

$$\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_c} = U_{3,\pm}(e_{92}, n, \eta_g) + iU_{4,\pm}(e_{92}, n, \eta_g) \tag{10.3}$$

The ES becomes

$$\overline{n}_{02D} = \zeta_1[1 + \exp(-\eta_{92})]^{-1} \tag{10.4}$$

where, $\overline{n}_{02D} = \zeta_1[1 + \exp(-\eta_{92})]$ where, $\eta_{92} = [E_{F92HD2D} - e_{92}](k_BT)^{-1}$.

(b) **Two-Band Model of Kane**

The HDDR becomes [35]

$$\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_c} = \gamma_2(e_{93}, \eta_g) - \left(n + \frac{1}{2}\right)\hbar\omega_0 \pm \frac{1}{2}g^*\mu_0 B \tag{10.5}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{93})]^{-1} \tag{10.6}$$

where, $\eta_{93} = [E_{F93HD2D} - e_{93}](k_BT)^{-1}$

(c) **Parabolic Energy Bands**

The HDDR becomes [35]

$$\frac{\hbar^2(\frac{n_z\pi}{d_z})^2}{2m_c} = \gamma_3(e_{94}, \eta_g) - (n + \frac{1}{2})\hbar\omega_0 \pm \frac{1}{2}g^*\mu_0 B \tag{10.7}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{94})]^{-1} \tag{10.8}$$

where, $\eta_{94} = [E_{F94HD2D} - e_{94}](k_BT)^{-1}$

(d) **The Model of Stillman et. al.**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_7(e_{95}, n, \eta_g) \tag{10.9}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{95})]^{-1} \tag{10.10}$$

where, $\eta_{95} = [E_{F95HD2D} - e_{95}](k_BT)^{-1}$

(e) **The Model of Palik et al.**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = A_{35,\pm}(e_{96}, n, \eta_g) \tag{10.11}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{96})]^{-1} \tag{10.12}$$

where, $\eta_{96} = [E_{F96HD2D} - e_{96}](k_BT)^{-1}$

10.2.3 The II-VI Semiconductors

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{8\pm}(e_{97}, n, \eta_g) \tag{10.13}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{97})]^{-1} \tag{10.14}$$

where, $\eta_{97} = [E_{F97HD2D} - e_{97}](k_BT)^{-1}$

10.2.4 The IV-VI Semiconductors

(a) **Cohen Model**

The HDDR becomes [35]

$$(\frac{n_z\pi}{d_z})^2 = U_{16\pm}(e_{98}, n, \eta_g) \tag{10.15}$$

where, $\eta_{98} = [E_{F98HD2D} - e_{98}](k_BT)^{-1}$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{98})]^{-1} \tag{10.16}$$

(b) **Lax Model**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{17\pm}(e_{99}, n, \eta_g) \tag{10.17}$$

where, $\eta_{99} = [E_{F99HD2D} - e_{99}](k_BT)^{-1}$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{99})]^{-1} \tag{10.18}$$

(c) **Dimmock Model**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{170}(e_{100}, n, \eta_g) \tag{10.19}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{100})]^{-1} \tag{10.20}$$

where, $\eta_{100} = [E_{F100HD2D} - e_{100}](k_BT)^{-1}$

(d) **Model of Bangert and Kastner**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{18}(e_{101}, n, \eta_g) \tag{10.21}$$

The ES becomes

$$\overline{n}_{02D} = \zeta_1[1 + \exp(-\eta_{101})]^{-1} \tag{10.22}$$

where, $\overline{n}_{02D} = 2\zeta_1[1 + \exp(-\eta_{101})]$ where, $\eta_{101} = [E_{F101HD2D} - e_{101}](k_BT)^{-1}$

(e) **Model of Foley and Langenberg**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{19}(e_{102}, n, \eta_g) \tag{10.23}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{102})]^{-1} \tag{10.24}$$

where, $\eta_{102} = [E_{F102HD2D} - e_{102}](k_BT)^{-1}$.

10.2.5 The Stressed Kane-Type Semiconductor

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{41}(e_{103}, n, \eta_g)|_{\theta=0} \tag{10.25}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{103})]^{-1} \tag{10.26}$$

where, $\eta_{103} = [E_{F103HD2D} - e_{103}](k_BT)^{-1}$.

10.2.6 The Te

The HDDR is given by [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{42\pm}(e_{104}, n, \eta_g) \tag{10.27}$$

The ES can be expressed as

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{104})]^{-1} \tag{10.28}$$

where, $\eta_{104} = [E_{F104HD2D} - e_{104}](k_BT)^{-1}$.

10.2.7 The Gallium Phosphide

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{43}(e_{105}, n, \eta_g) \tag{10.29}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{105})]^{-1} \tag{10.30}$$

where, $\eta_{105} = [E_{F105HD2D} - e_{105}](k_BT)^{-1}$.

10.2.8 The Platinum Antimonide

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{44}(e_{106}, n, \eta_g) \tag{10.31}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{106})]^{-1} \tag{10.32}$$

where, $\eta_{106} = [E_{F106HD2D} - e_{106}](k_BT)^{-1}$.

10.2.9 The Bismuth Telluride

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{45}(e_{107}, n, \eta_g) \tag{10.33}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{107})]^{-1} \tag{10.34}$$

where, $\eta_{107} = [E_{F107HD2D} - e_{107}](k_BT)^{-1}$.

10.2.10 The Germanium

a. **Model of Cardona et al.**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{46}(e_{108}, n, \eta_g) \tag{10.35}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{108})]^{-1} \tag{10.36}$$

where, $\eta_{108} = [E_{F108HD2D} - e_{108}](k_BT)^{-1}$.

b. **Model of Wang and Ressler**

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{47}(e_{109}, n, \eta_g) \tag{10.37}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{109})]^{-1} \tag{10.38}$$

where, $\eta_{109} = [E_{F109HD2D} - e_{109}](k_BT)^{-1}$.

10.2.11 The Gallium Antimonide

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{48}(e_{110}, n, \eta_g) \tag{10.39}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{110})]^{-1} \tag{10.40}$$

where, $\eta_{110} = [E_{F110HD2D} - e_{110}](k_BT)^{-1}$.

10.2.12 The II-V Materials

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{49\pm}(e_{111}, n, \eta_g) \tag{10.41}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{111})]^{-1} \tag{10.42}$$

where, $\eta_{111} = [E_{F111HD2D} - e_{111}](k_BT)^{-1}$.

10.2.13 The Lead Germanium Telluride

The HDDR becomes [35]

$$\left(\frac{n_z\pi}{d_z}\right)^2 = U_{50\pm}\left(e_{112}, n, \eta_g\right) \tag{10.43}$$

The ES becomes

$$n_{02D} = \zeta_1[1 + \exp(-\eta_{112})]^{-1} \tag{10.44}$$

where, $\eta_{112} = [E_{F112HD2D} - e_{112}](k_BT)^{-1}$.

10.3 Summary and Conclusion

Under magneto-size quantization, all directions are being quantized and the variation of any electronic property is much more prominent as compared with the same in QWs and NWs, respectively. We have not given any plot of ES with respect to any variable in the present case since we believe our able readers will plot the same and enjoy intricate physics, computational analysis and advanced mathematics and will create new problems both theoretical and experimental.

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Chapter 11
The HUP and the ES in Heavily Doped Non-parabolic Quantum Wells (HDQWs) Under Cross-Fields Configuration

11.1 Introduction

The cross fields electron transport has been widely studied in the literature [1–14] and here we investigate the ES in heavily doped ultrathin films under $E_0 \perp B$ chronologically for all the cases of this chapter. Section 11.3 explores the summary and conclusion.

11.2 Mathematical Basis

11.2.1 The Nonlinear Optical Semiconductors

The HDDR becomes

$$T_{22}(p_1) = \left[\left(\left(n+\frac{1}{2}\right)\hbar\omega_{01}\right) + \left(\frac{[\hbar]^2}{2a(p_1)}\right)\left(\frac{\pi n_z}{d_z}\right)^2 - \left(\frac{E_0\hbar k_y \rho(p_1)}{B}\right) - \left(\frac{M_\perp \rho^2(\rho_1)E_0^2}{2B^2}\right)\right] \tag{11.1}$$

where, $p_1 = (E, \eta_g)$.

Using Eq. (11.1), the EEM becomes

$$m_y^*(p_2) = R.P.(B/E_0)T_{49}(p_3)[T_{49}(p_3)]' \tag{11.2}$$

where, $p_2 = (e_{fA1}, \eta_g, n, E_0, n_z)$ and $p_3 = (e_{fA1}, \eta_g, n_z)$.

The n_s becomes

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$$n_s = R.P.\overline{n}_s \tag{11.3}$$

where,

$$\overline{n}_s = 2\zeta_1[1+\exp(-\eta_{11,1})]^{-1} \tag{11.4}$$

In which $\eta_{11,i} = (e_{fAi} - E_{11.i})(k_B T)^{-1}$, e_{fAi} is the Fermi energy in the present case, $i = 1, 2, 3, 4, 5, 6$ and 7, $E_{11,i}$ is the sub-band energy which can be obtained from the respective HDDR by substituting $E = E_{11,j}$ and $k_y = 0$.

11.2.2 The Kane-Type III-V Semiconductors

(a) Using the constraints $\delta = 0$, $\Delta_{||} = \Delta_{\perp} = \Delta$ and $m^*_{||} = m^*_{\perp} = m_c$, Eq. (11.1) becomes

$$T_{33}(p_1) = \left(n+\frac{1}{2}\right)\hbar\omega_{01} + \frac{\left[\hbar\frac{\pi n_z}{d_z}\right]^2}{2m_c} - \frac{E_0}{B}\hbar k_y\{T_{33}(p_1)\}' - \frac{m_c E_0^2[\{T_{33}(p_1)\}']^2}{2B^2} \tag{11.5}$$

From Eq. (11.5), the EEM becomes

$$m^*_y(p_2) = R.P.\,(B/E_0)^2 T_{50}(p_3)[T_{50}(e_{fA2},\eta_g,n_z)]' \tag{11.6}$$

where,

$$T_{50}(e_{fA2},\eta_g,n_z) = \left[T_{30}(e_{fA2},\eta_g) - \left(n+\frac{1}{2}\right)\hbar\omega_0 - \frac{\left[\hbar\frac{\pi n_z}{d_z}\right]^2}{2m_c} + \frac{m_\perp E_0^2[\{T_{33}(e_{fA2},\eta_g)\}']^2}{2B^2}\right]\left[\{T_{33}(e_{fA2},\eta_g)\}'\right]^{-1}$$

The n_s becomes

$$\overline{n}_s = 2\zeta_1[1+\exp(-\eta_{11,2})]^{-1} \tag{11.7}$$

where,

$$\overline{n}_s = 2\zeta_1[1 + \exp(-\eta_{11,2})] \tag{11.8}$$

In which $\eta_{11,2} = (e_{fA2} - E_{11.2})(k_B T)^{-1}$

(b) **HD Two-Band Model of Kane**

Using the constraints $\Delta >> E_g$, Eq. (11.5) becomes

$$\gamma_2(p_1) = \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{E_0}{B}\hbar k_y \gamma_2'(p_1) - \frac{m_c E_0^2}{2B^2}(\gamma_2'(p_1)) + \frac{\left[\hbar\frac{n_z\pi}{d_z}\right]^2}{2m_c} \tag{11.9}$$

Using Eq. (11.9), the EEM becomes

$$m_y^*(e_{fA3}, \eta_g, n, E_0, n_z) = (B/E_0)^2 T_{51}(e_{fA3}, \eta_g, n_z) \left[T_{51}(e_{fA3}, \eta_g, n_z)\right]' \tag{11.10}$$

where

$$T_{51}(e_{fA3}, \eta_g, n_z) = \left[\gamma_2(e_{fA3}, \eta_g) - \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{\left[\hbar\frac{\pi n_z}{d_z}\right]^2}{2m_c} + \frac{m_c E_0^2\left[\{\gamma_2(e_{fA3}, \eta_g)\}'\right]^2}{2B^2}\right] \left[\{\gamma_2(e_{fA3}, \eta_g)\}'\right]^{-1}$$

The n_s becomes

$$n_s = \overline{n}_s \tag{11.11}$$

where,

$$\overline{n}_s = 2\zeta_1[1 + \exp(-\eta_{11,3})]^{-1} \tag{11.12}$$

In which $\eta_{11,3} = (e_{fA3} - E_{11.3})(k_B T)^{-1}$.

(c) **HD Parabolic Energy Bands**

Under this condition $\alpha \to 0$, Eq. (11.9) becomes

$$\gamma_3(p_1) = \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{E_0}{B}\hbar k_y \gamma_3'(p_1)$$

$$-\frac{m_c E_0^2}{2B^2}\left(\gamma_3'(p_1)\right)^2+\frac{\left[\hbar\frac{n_z\pi}{d_z}\right]^2}{2m_c} \tag{11.13}$$

From Eq. (11.13) the EEM becomes

$$m_y^*(e_{fA4},\eta_g,n,E_0,n_z)=(B/E_0)^2T_{52}(e_{fA4},\eta_g,n_z)\left[T_{52}(e_{fA4},\eta_g,n_z)\right]' \tag{11.14}$$

where

$$T_{52}(e_{fA4},\eta_g,n_z)=\left[\gamma_2(e_{fA4},\eta_g)-\left(n+\frac{1}{2}\right)\hbar\omega_0-\frac{\left[\hbar\frac{\pi n_z}{d_z}\right]^2}{2m_c}+\frac{m_cE_0^2[\{\gamma_3(e_{fA4},\eta_g)\}']^2}{2B^2}\right]\left[\{\gamma_3(e_{fA4},\eta_g)\}'\right]^{-1}$$

The n_s becomes

$$n_s=\overline{n}_s \tag{11.15}$$

where,

$$\overline{n}_s=2\zeta_1[1+\exp(-\eta_{11,4})]^{-1} \tag{11.16}$$

In which $\eta_{11,4}=(e_{fA4}-E_{11.4})(k_BT)^{-1}$.

11.2.3 The II-VI Semiconductors

The HDDR becomes

$$\gamma_3(p_1)=\beta_1(n,E_0)-\frac{E_0}{B}\hbar k_y\gamma_3'(p_1)-\frac{m_{||}^*E_0^2}{2B^2}(\gamma_3'(p_1))^2+\frac{\left[\hbar\frac{n_z\pi}{d_z}\right]^2}{2m_{||}} \tag{11.17}$$

Using Eq. (11.17), the EEM becomes

$$m_y^*(e_{fA4}, \eta_g, n, E_0, n_z) = (B/E_0)^2 T_{53}(e_{fA5}, \eta_g, n_z) \left[T_{53}(e_{fA5}, \eta_g, n_z)\right]' \tag{11.18}$$

where

$$T_{53}(e_{fA5}, \eta_g, n_z) = \left[\gamma_3(e_{fA5}, \eta_g) - \beta_1(n, E_0) - \frac{[\hbar\frac{\pi n_z}{d_z}]^2}{2m_{||}^*} + \frac{m_{||}^* E_0^2 [\{\gamma_3(e_{fA5}, \eta_g)\}']^2}{2B^2}\right] [\{\gamma_3(e_{fA5}, \eta_g)\}']^{-1}$$

The n_s become

$$n_s = \overline{n}_s \tag{11.19}$$

where,

$$\overline{n}_s = 2\zeta_1[1 + \exp(-\eta_{11,5})]^{-1}] \tag{11.20}$$

In which $\eta_{11,5} = (e_{fA5} - E_{11.5})(k_B T)^{-1}$.

11.2.4 The IV-VI Semiconductors

The HDDR becomes

$$g^*(p_1) = \left(n + \frac{1}{2}\right)\hbar\overline{\omega_{i1}}(p_1) + \frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2M_3^*(p_1)} - \frac{E_0}{B}\rho_1^*(p_1)\hbar k_y - \frac{E_0^2}{2B^2}\left[\rho_1^*(p_1)\right]^2 M_1^*(p_1) \tag{11.21}$$

Using Eq. (11.21), the EEM becomes

$$m_y^*(e_{fA6}, \eta_g, n, E_0, n_z) = R.\,P.\,(B/E_0)^2 T_{54}(e_{fA6}, \eta_g, n_z)[T_{54}(e_{fA6}, \eta_g, n_z)]' \tag{11.22}$$

where

$$T_{54}(e_{fA6}, \eta_g, n_z) = \left[g^*(e_{fA6}, \eta_g)\left(n + \frac{1}{2}\right)\hbar\overline{\omega_{i1}}(e_{fA6}, \eta_g)\right.$$

$$+\frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2M_3^*(e_{fA6},\eta_g)}-\frac{E_0^2}{2B^2}\left[\rho_1^*(e_{fA6},\eta_g)\right]^2 M_1^*(e_{fA6},\eta_g)\Big]\left[\rho_1^*(e_{fA6},\eta_g)\right]^{-1}$$

The n_s becomes

$$n_s = R.P.\overline{n}_s \tag{11.23}$$

where,

$$\overline{n}_s = 2\zeta_1[1+\exp(-\eta_{11,6})]^{-1} \tag{11.24}$$

In which $\eta_{11,6} = (e_{fA6} - E_{11.6})(k_BT)^{-1}$.

11.2.5 The Stressed Kane-Type Semiconductors

The HDDR becomes

$$G^*(p_1) = \left(n+\frac{1}{2}\right)\hbar\overline{\omega_i}(p_1) + \frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_3^*(p_1)} - \frac{E_0}{B}\rho^*(p_1)\left[\frac{m_1^*(p_1)}{m_2^*(p_1)}\right]^{1/2}\hbar k_y - \frac{E_0^2}{2B^2}\left[\rho^*(p_1)\right]^2 m_1^*(p_1) \tag{11.25}$$

Using Eq. (11.25), the EEM becomes

$$m_y^*(e_{fA7},\eta_g,n,E_0,n_z) = R.P.(B/E_0)^2 T_{55}(e_{fA7},\eta_g,n_z)\left[T_{55}(e_{fA7},\eta_g,n_z)\right]' \tag{11.26}$$

where

$$T_{55}(e_{fA7},\eta_g,n_z) = \Bigg[G^*(e_{fA7},\eta_g) = \left(n+\frac{1}{2}\right)\hbar\overline{\omega_i}(e_{fA7},\eta_g) + \frac{\hbar^2\left(\frac{n_z\pi}{d_z}\right)^2}{2m_3^*(e_{fA7},\eta_g)} - \frac{E_0^2}{2B^2}\left[\rho^*(e_{fA7},\eta_g)\right]m_1^*(e_{fA7},\eta_g)$$

$$-\frac{E_0^2}{2B^2}\left[\rho^*(E,\eta_g)\right]^2 m_1^*(E,\eta_g)\Bigg]\left[m_4^*(e_{fA7},\eta_g)\right]^{-1}$$

The n_s becomes

$$n_s = R.P.\overline{n}_s \tag{11.27}$$

where,

$$\overline{n}_s = 2\zeta_1[1+\exp(-\eta_{11,7})]^{-1} \tag{11.28}$$

In which $\eta_{11,7} = (e_{fA7} - E_{11.7})(k_B T)^{-1}$.

11.3 Summary and Conclusion

The DRs are quantized straight lines and depend on ES.

- The cross fields introduces energy and quantum number dependent mass anisotropy.

We have not given any plot of ES in this case since we believe our able readers will plot the same and enjoy intricate physics, computational analysis and advanced mathematics and will create new problems both theoretical and experimental.

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Chapter 12
The HUP and the ES in Doping Super Lattices of HD Non-parabolic Semiconductors Under Magnetic Quantization

12.1 Introduction

The physical properties of doping superlattices (DSL) have extensively investigated [1–31]. In Sect. 12.2, we investigate the ES in DSLs chronologically as given in the content of this chapter. Section 12.3 contains the result and discussions.

12.2 Mathematical Basis

12.2.1 The Nonlinear Optical Semiconductors

The magneto DR becomes [31]

$$\frac{\left(n_i+\frac{1}{2}\right)}{\hbar T_{21}(E_{10,1},\eta_g)}\omega_{8\mathrm{HD}}(E_{10,1},\eta_g)+\frac{\hbar eB\left(n_i+\frac{1}{2}\right)}{m_{\perp}^{*}T_{22}(E_{10,1},\eta_g)}=1 \tag{12.1}$$

where $E_{10,1}$ is the totally quantized energy.

The ES becomes

$$n_{02D}=R.P.(\overline{n}_{02D}) \tag{12.2}$$

where,

$$\overline{n}_{02D}=\zeta_2[1+\exp(-\eta_{12,1})]^{-1} \tag{12.3}$$

$\zeta_2=\frac{g_v eB}{\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{n_z=1}^{n_{z\max}}$, $\eta_{12,1}=[E_{F12\mathrm{HD}2D}-E_{10,1}](k_BT)^{-1}$ and $E_{F12\mathrm{HD}2D}$ is the corresponding Fermi energy.

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When $\eta_g \to 0$, the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \tag{12.4}$$

where,

$$\overline{n}_{02D} = \zeta_2[1 + \exp(-\eta_{12,2})]^{-1} \tag{12.5}$$

$\eta_{12,2} = [E_{F122D} - E_{10,2}](k_B T)^{-1}$ and E_{F122D} is the corresponding Fermi energy.

12.2.2 The Kane-Type III-V Semiconductors

(a) The magneto DR is given by Ghatak [31]

$$\left(n + \frac{1}{2}\right)\hbar\omega_0 = \left[T_{31}(E_{10,3}, \eta_g) + iT_{32}(E_{10,3}, \eta_g) - \left(n_i + \frac{1}{2}\right)\hbar\omega_{9\mathrm{HD}}(E_{10,3}, \eta_g)\right] \tag{12.6}$$

where, $E_{10,3}$ is the totally quantized energy.

The ES can be written as

$$n_{02D} = R.P.(\overline{n}_{02D}) \tag{12.7}$$

where,

$$\overline{n}_{02D} = \zeta_2[1 + \exp(-\eta_{12,3})]^{-1} \tag{12.8}$$

and $\eta_{12,3} = [E_{F12\mathrm{HD}2D} - E_{10,3}](k_B T)^{-1}$.

When $\eta_g \to 0$, we can write that the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \tag{12.9}$$

where

$$\overline{n}_{02D} = \zeta_2[1 + \exp(-\eta_{12,4})]^{-1} \tag{12.10}$$

and $\eta_{12,4} = [E_{F122D} - E_{10,4}](k_B T)^{-1}$.

(b) The magneto DR is given by Ghatak [31]

$$\left(n+\frac{1}{2}\right)\hbar\omega_0 = \left[\gamma_2(E_{10,5},\eta_g) - \left(n_i+\frac{1}{2}\right)\hbar\omega_{10\mathrm{HD}}(E_{10,5},\eta_g)\right] \quad (12.11)$$

The ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \quad (12.12)$$

where,

$$\overline{n}_{02D} = \zeta_2[1+\exp(-\eta_{12,5})]^{-1} \quad (12.13)$$

and $\eta_{12,5} = [E_{F12\mathrm{HD}2D} - E_{10,5}](k_BT)^{-1}$.

When $\eta_g \to 0$, we can write the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \quad (12.14)$$

where,

$$\overline{n}_{02D} = \zeta_2[1+\exp(-\eta_{12,6})]^{-1} \quad (12.15)$$

and $\eta_{12,6} = [E_{F122D} - E_{10,6}](k_BT)^{-1}$.

(c) The magneto DR is given by Ghatak [31]

$$\left(n+\frac{1}{2}\right)\hbar\omega_0 = \left[\gamma_3(E_{10,7},\eta_g) - \left(n+\frac{1}{2}\right)\hbar\omega_{11\mathrm{HD}}(E_{10,7},\eta_g)\right] \quad (12.16)$$

The ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \quad (12.17)$$

where,

$$\overline{n}_{02D} = \zeta_2[1+\exp(-\eta_{12,7})]^{-1} \quad (12.18)$$

and $\eta_{12,7} = [E_{F12\mathrm{HD}2D} - E_{10,7}](k_BT)^{-1}$.

When $\eta_g \to 0$, we can write the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \quad (12.19)$$

where,

$$\overline{n}_{02D} = \zeta_2[1 + \exp(-\eta_{12,8})]^{-1} \tag{12.20}$$

and $\eta_{12,8} = [E_{F12HD2D} - E_{10,8}](k_B T)^{-1}$.

12.2.3 The II-VI Semiconductors

The magneto DR is given by Ghatak [31]

$$\gamma_3(E_{10,9}, \eta_g) = \left[a_0' \frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right] + \left(n_i + \frac{1}{2}\right)\hbar\omega_{30\mathrm{HD}}(E_{10,9}, \eta_g) \pm \overline{\lambda}_0\sqrt{\left[\frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right]} \tag{12.21}$$

The ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \tag{12.22}$$

where,

$$\overline{n}_{02D} = \frac{\zeta_2}{2}[1 + \exp(-\eta_{12,9})]^{-1} \tag{12.23}$$

and $\eta_{12,9} = [E_{F12\mathrm{HD}2D} - E_{10,9}](k_B T)^{-1}$.

When $\eta_g \to 0$, we can write the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \tag{12.24}$$

where,

$$\overline{n}_{02D} = \frac{\zeta_2}{2}[1 + \exp(-\eta_{12,10})]^{-1} \tag{12.25}$$

and $\eta_{12,10} = [E_{F12HD2D} - E_{10,10}](k_B T)^{-1}$.

12.2.4 The IV-VI Semiconductors

The magneto DR is given by [3]

$$\frac{2eB}{\hbar}\left(n+\frac{1}{2}\right)=\delta_{15}(E_{10,11},\eta_g,n_i) \tag{12.26}$$

The ES can be written as

$$n_{02D}=(\overline{n}_{02D}) \tag{12.27}$$

where,

$$\overline{n}_{02D}=\zeta_2[1+\exp(-\eta_{12,11})]^{-1} \tag{12.28}$$

and $\eta_{12,11}=[E_{F12\mathrm{HD}2D}-E_{10,11}](k_BT)^{-1}$.

When $\eta_g\rightarrow 0$, we can write the ES can be written as

$$n_{02D}=(\overline{n}_{02D}) \tag{12.29}$$

where,

$$\overline{n}_{02D}=\zeta_2[1+\exp(-\eta_{12,12})]^{-1} \tag{12.30}$$

and $\eta_{12,12}=[E_{F12HD2D}-E_{10,12}](k_BT)^{-1}$.

12.2.5 The Stressed Kane-Type Semiconductors

The magneto DR in this case is given by Ghatak [31]

$$\left(n+\frac{1}{2}\right)\hbar\omega_{90\mathrm{HD}}(E_{10,13},\eta_g)+S_{11}(E_{10,13},\eta_g)\delta_{19}(E_{10,13},\eta_g,n_i) \tag{12.31}$$

The ES can be written as

$$n_{02D}=(\overline{n}_{02D}) \tag{12.32}$$

where,

$$\overline{n}_{02D}=\zeta_2[1+\exp(-\eta_{12,13})]^{-1} \tag{12.33}$$

and $\eta_{12,13}=[E_{F12\mathrm{HD}2D}-E_{10,13}](k_BT)^{-1}$.

When $\eta_g\rightarrow 0$, we can write the ES can be written as

$$n_{02D} = (\overline{n}_{02D}) \tag{12.34}$$

where,

$$\overline{n}_{02D} = \zeta_2[1 + \exp(-\eta_{12,14})]^{-1} \tag{12.35}$$

and $\eta_{12,14} = [E_{F12HD2D} - E_{10,14}](k_B T)^{-1}$.

12.3 Result and Discussion

Taking various types of HD NIPI structures as given in [31], we have plotted the E_F as functions of different physical variables as show in Figs. 12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 12.7, 12.8, 12.9, 12.10 and 12.11, and we note the following features:

1. From Figs. 12.1, 12.2, 12.3, 12.4 and 12.5, we note that the E_F oscillates with B^{-1}.
2. From Figs. 12.6, 12.7, 12.8, 12.9, 12.10 and 12.11, we note that the E_F changes with n_0 in periodic manner.

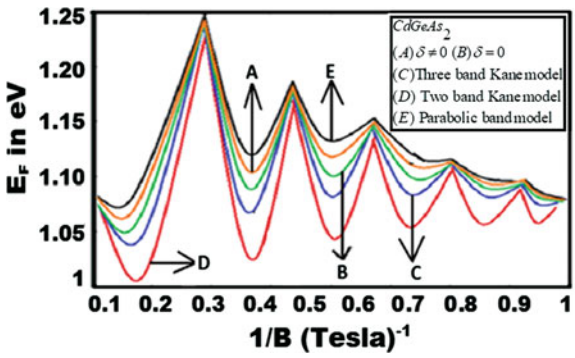

Fig. 12.1 E_F against B^{-1} for $CdGeAs_2$ HD DSLs

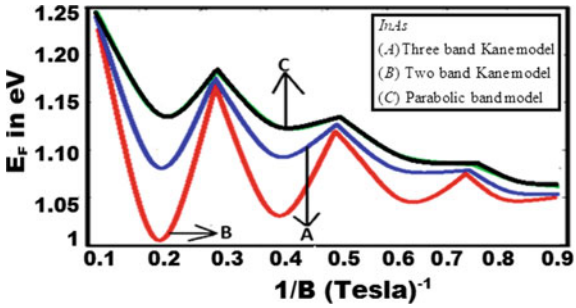

Fig. 12.2 E_F against B^{-1} for InAs HD DSLs

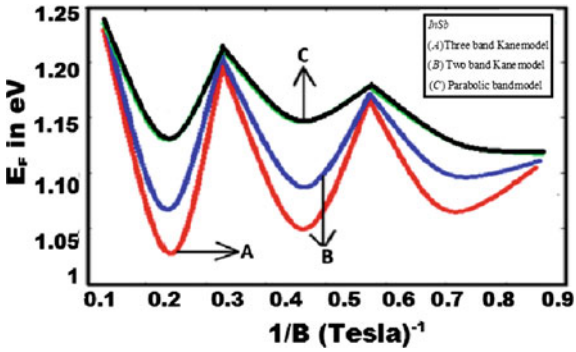

Fig. 12.3 E_F against B^{-1} for InSb HD DSLs

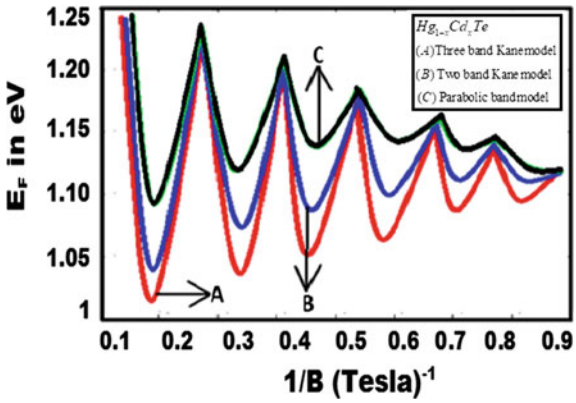

Fig. 12.4 E_F against B^{-1} for $Hg_{1-x}Cd_xTe$ HD DSLs

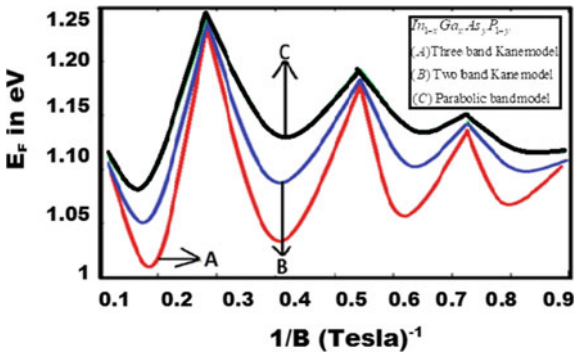

Fig. 12.5 E_F against B^{-1} for $In_{1-x}Ga_xAs_yP_{1-y}$ HD DSLs

Finally, we conclude that this chapter shows basic features of the magneto Fermi energy in HDDSLs of various band structures.

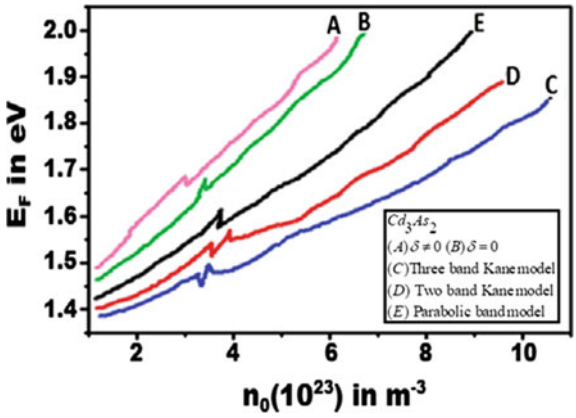

Fig. 12.6 E_F against n_0 for Cd_3As_2 HD DSLs

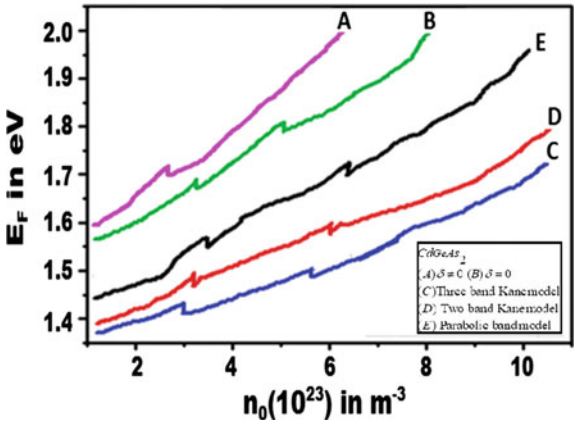

Fig. 12.7 E_F against n_0 for $CdGeAs_2$ HD DSLs

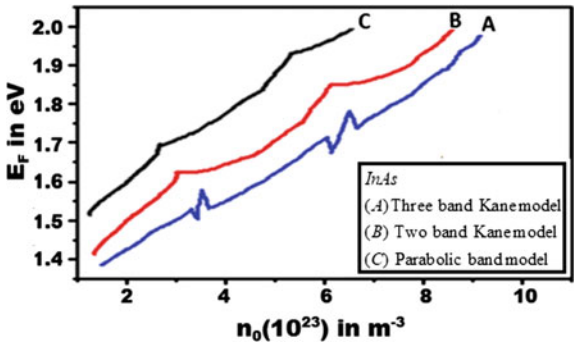

Fig. 12.8 E_F against n_0 for InAs HD DSLs

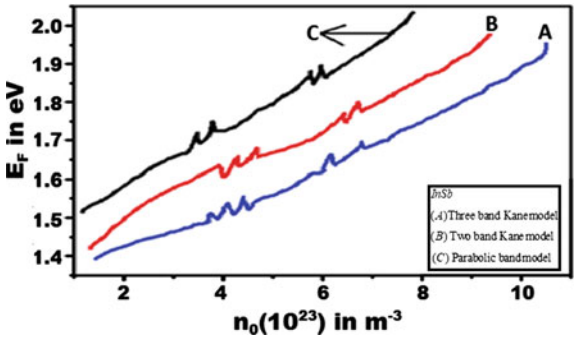

Fig. 12.9 E_F against n_0 forInSb HD DSLs

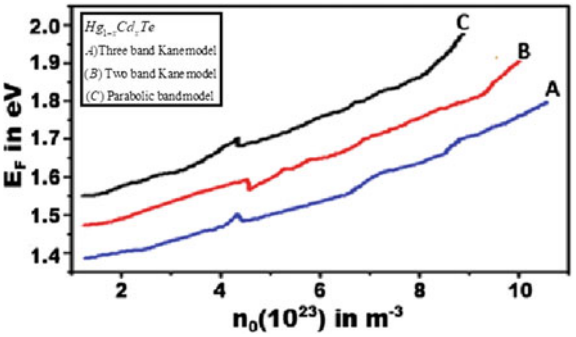

Fig. 12.10 E_F against n_0 for $Hg_{1-x}Cd_xTe$ HD DSLs

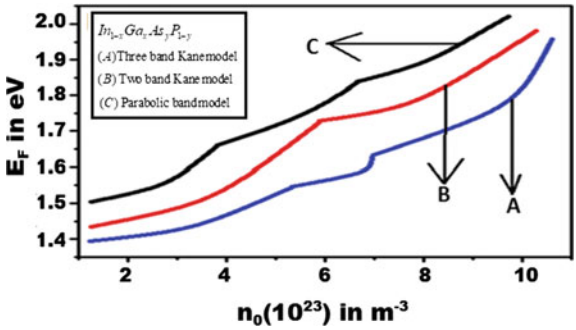

Fig. 12.11 E_F against n_0 for $In_{1-x}Ga_xAs_yP_{1-y}$ HD DSLs

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Chapter 13
The HUP and Magneto ES in Accumulation Layers

13.1 Introduction

The electronic properties of the magneto accumulation have relatively being less studied in the literature [1–25]. In the last chapter in Sect. 13.2, we shall study the magneto ES of the same in accordance with the chronological content of this chapter, and Sect. 13.3 represents the summary and conclusion in this context.

13.2 Mathematical Basis

13.2.1 The Nonlinear Optical Semiconductors

The HDDR in magneto accumulation layers becomes [25]

$$\left(n+\frac{1}{2}\right)\frac{\hbar eB}{m_{||}^{*}}=L_6(E_{13,1},i,\eta_g) \tag{13.1}$$

The ES becomes

$$n_s=R.\ P.\ \overline{n}_s \tag{13.2}$$

where,

$$\overline{n}_s=\zeta_4[1+\exp(-\eta_{13,1})]^{-1}, \tag{13.3}$$

$\zeta_4=\frac{g_v eB}{\pi\hbar}\sum_{n=0}^{n_{\max}}\sum_{i=0}^{n_{i\max}}$ and $\eta_{13,1}=(E_{FHD2D13}-E_{13,1})(k_BT)^{-1}$.

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13.2.2 The III-V and Opto-electronic Materials

(a) The HDDR in magneto accumulation layers becomes [25]

$$T_{90}(E_{13,3}, \eta_g) = \left(n + \frac{1}{2}\right)\frac{\hbar e B}{m_c} + S_i\left[\frac{\hbar|e|F_s[T_{90}(E_{13,3}, \eta_g)]'}{\sqrt{2m_c}}\right]^{2/3} \tag{13.4}$$

The ES becomes

$$n_s = R.\ P.\ \overline{n}_s \tag{13.5}$$

where,

$$\overline{n}_s = \zeta_4[1 + \exp(-\eta_{13,3})]^{-1}, \tag{13.6}$$

$$\eta_{13,3} = (E_{FHD2D13} - E_{13,3})(k_B T)^{-1}$$

13.2.3 The II-VI Semiconductors

The HDDR in magneto accumulation layers becomes [25]

$$\gamma_3(E_{13,9}, \eta_g) = a'\left[{}_0\frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right] \pm \overline{\lambda}_0\left[\frac{2eB}{\hbar}\left(n + \frac{1}{2}\right)\right]^{1/2} + S\left[{}_i\frac{\hbar|e|F_s\gamma_3'(E_{13,9}, \eta_g)}{\sqrt{2m_{||}^*}}\right]^{2/3} \tag{13.7}$$

The ES becomes

$$n_s = \overline{n}_s \tag{13.8}$$

where,

$$\overline{n}_s = (\frac{\zeta_4}{2})[1 + \exp(-\eta_{13,9})]^{-1} \tag{13.9}$$

$$\eta_{13,9} = (E_{FHD2D13} - E_{13,9})(k_B T)^{-1}$$

13.2.4 The IV-VI Semiconductors

The HDDR in magneto accumulation layers becomes [25]

$$\left(n+\frac{1}{2}\right)\frac{eB}{\hbar}\sqrt{\theta_1(E_{13,11},i,\eta_g)\theta_2(E_{13,11},i,\eta_g)}=\theta_3(E_{13,11},i,\eta_g) \tag{13.10}$$

The ES becomes

$$n_s = R.\ P.\overline{n}_s \tag{13.11}$$

where,

$$\overline{n}_s=\zeta_4[1+\exp(-\eta_{13,11})]^{-1} \tag{13.12}$$

$$\eta_{13,11} = (E_{FHD2D13}-E_{13,11})(k_BT)^{-1}$$

13.2.5 The Stressed Kane-Type Semiconductors

The HDDR in magneto accumulation layers becomes [25]

$$\left(n+\frac{1}{2}\right)\frac{eB}{\hbar}\left[\theta_{13}(E_{13,13},i,\eta_g)\theta_{23}(E_{13,13},i,\eta_g)\right]^{1/2}=\theta_{33}(E_{13,13},i,\eta_g) \tag{13.13}$$

The ES becomes

$$n_s = \overline{n}_s \tag{13.14}$$

where,

$$\overline{n}_s=\zeta_4[1+\exp(-\eta_{13,13})]^{-1} \tag{13.15}$$

$$\eta_{13,13} = (E_{FHD2D13}-E_{13,13})(k_BT)^{-1}$$

13.2.6 The Germanium

The HDDR in magneto accumulation layers becomes [25]

$$\left(n+\frac{1}{2}\right)\frac{\hbar eB}{\sqrt{m_1^* m_2^*}} = \gamma_{10}(E_{13,15}, i, \eta_g) \tag{13.16}$$

The ES becomes

$$n_s = \overline{n}_s \tag{13.17}$$

where,

$$\overline{n}_s = \zeta_4[1+\exp(-\eta_{13,15})]^{-1} \tag{13.18}$$

$$\eta_{13,15} = (E_{FHD2D13} - E_{13,15})(k_B T)^{-1}$$

13.3 Summary and Conclusion

- It is the band structure which changes in a fundamental way and consequently all the physical properties of the magneto accumulation layers.

The importance of magneto accumulation layers is already well-known in the field of low dimensional materials, and we hope that our readers are in tune with the same. We have not given any plot of ES in this case since we believe our able readers will plot the same and enjoy intricate physics, computational analysis and advanced mathematics and will create new problems both theoretical and experimental.

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